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EDWIN DUNKIN, Esq., F.R.S., President, in the Chair.

Robert Stuart Callcott, 1 Bath Place, Kensington, W.; and Samuel Johnson, M.B. (Edin.), 23 Gloucester Terrace, Hyde Park, W.;

were balloted for and duly elected Fellows of the Society.

Notes on Mr. Stone's Explanation of the Errors of Hansen's Lunar Tables. By Professor Simon Newcomb.

The courtesy afforded me by the Royal Astronomical Society in publishing my questions respecting Mr. Stone's theory, which appeared in the March number of its *Notices*, leads me to enter a little more fully into the case, and to show the very important bearing which those questions have upon the applicability of the theory.

Mr. Stone is understood to maintain that no one has answered his arguments. To answer an argument one must first understand it; and I frankly confess that this is more than I have been able to do. In particular I have been entirely unable to master the reasonings and equations found on p. 336 of vol. xliii. and p. 226, vol. xliv. But the value of an argument depends largely upon its conclusions; and it is the defect in this respect which I wish to point out.

So far as Hansen's Tables are concerned Mr. Stone has not even attempted to make out a logical case. This will be made clear by showing what his conclusion is, and how alone it can be proved. I understand the conclusion to be—

"That the increasing errors in Hansen's Tables of the Moon since 1864 arise from the substitution of Le Verrier's Solar Tables for Carlini's in the British *Nautical Almanac* for 1864 and subsequent years."

Now the logical process for establishing this proposition are clear and simple, and may be stated as follows :—

A. Discover what the apparent errors of the Lunar Tables would have been had Carlini's Solar Tables been continued in use until the present time.

B. Show what the errors actually are under the influence of Le Verrier's Tables. The difference of these results will show the effect of the change of tables.

C. Determine what the theoretical errors are arising from the cause Mr. Stone has discovered.

The proof would then consist in showing that C corresponded to B minus A.

Now, Mr. Stone has made no attempt to cover more than one of these three points. He gives B, the actual errors of the Lunar Tables, very fully, on p. 337 of vol. xliii. But he does not profess to have made any calculation showing what these errors would have been had Carlini's Tables been continued. This is the first logical defect. Secondly, he does not state, except in algebraic form and with an unknown symbol t, what the difference would be according to his theory.

Thus of the two terms of comparison B−A and C, whose agreement is necessary to the *prima facie* establishment of his theory, he gives neither.

Not only has Mr. Stone suffered these two fatal defects in his argument to stand a year, but he has resisted all my appeals to give a general definition of his symbol t, which shall apply to any ephemeris whatever, and thus enable anyone to calculate the result of his theory.

Now, were I to attempt to apply Mr. Stone's theory in what would seem the most natural way, I should count the time t from the epoch 1728, when the two tables agreed in mean longitude. This would make a deviation at the present time of nearly two minutes, whereas the actual errors of the tables are scarcely more than one-tenth of this quantity. If this interpretation is correct the theory could be established only by showing that the errors of the Lunar Tables are two minutes different from what they would have been had not the change of solar tables been made.

Apparently, though Mr. Stone nowhere explicitly says so, he desires the computer to count the epoch t from the year 1864, when the new tables were introduced into the British *Nautical Almanac*. Now, a large number of observations of the Moon have been compared with the data of the American Ephemeris, in which Hansen's Solar Tables, the mean motion of which is substantially identical with those of Le Verrier, were introduced in 1858. If I adopt the epoch 1858 the result will be a correction different by $4''.5$ from that applicable on the same theory to the British *Nautical Almanac*. But, as a matter of fact, all the

numerical data of the two ephemerides are substantially identical; so that the formula must be the same for both. If, on the other hand, I take the year 1864 as the epoch for the American Ephemeris, I proceed on the absurd theory that the results of observations and computations made in a country 3,000 miles away are now several seconds different from what they would have been had Le Verrier's Tables never been introduced into the British *Nautical Almanac.*

In a word, to count from any other epoch than 1728 involves the absurdity that the result of computing from a set of tables is a function of the time when some particular person began to use those tables. I have found this difficulty insurmountable. I think Mr. Stone will also find it insurmountable. Indeed, his refusal to make known so simple a matter as what the deviations of the lunar tables in the American Ephemeris ought to be on his theory seems to me tantamount to an admission that his theory cannot be applied to observations and computations made outside of England.

Addendum.

The following statement of the dilemma in attempting to apply Mr. Stone's theory may be more clear and explicit:—A comparison of Hansen's Lunar Tables with observation by means of the data of the American Ephemeris shows the same phenomena as in the comparison by means of the British *Nautical Almanac*—namely, a close agreement up to the year 1863, and from that epoch a steadily increasing deviation. The numbers are in fact substantially the same for both ephemerides. Hence the corrections must be the same for both. But if I adopt Mr. Stone's theory, and apply the same corrections, I implicitly assume that this deviation, which dates from 1863 in our ephemeris, is due to the adoption of new tables in another country. I do not see how to apply the theory without this absurdity.

Answers to Prof. Newcomb's Questions on the Changes in the Adopted Unit of Mean Time. By E. J. Stone, M.A., F.R.S.

I greatly regret to find that my views have not been explained with sufficient clearness to enable Prof. Newcomb to deal directly with my arguments. All I want is to have a most serious question rightly settled. If my views are right a new departure can be made in astronomy, and a considerable advance would be certain to follow. If my views are wrong it is most desirable that they should be so completely disproved that their revival at some future period may be effectually prevented, and that our attention should be directed to some other physical explanation of our difficulties.

That our estimations of the length of the tropical year

change *per saltum*, and that our Lunar Tables require complete readjustment to an exactly proportionate amount from the adoption of new Solar Tables, are facts whose connection and significance cannot safely be overlooked.

The agreement between Hansen's Tables and observation when the mean times are reduced to one uniform scale, in accordance with my views, is much closer, as a matter of fact, than that produced by the empirical corrections to Hansen's Tables introduced by Prof. Newcomb, although these corrections are based upon a discussion of recent observations with the introduction of at least four disposable constants to secure an agreement between the corrected tabular places and observation.

I have not the slightest difficulty in meeting Prof. Newcomb's dilemma in the most direct manner; but my way of meeting it will not be satisfactory to Prof. Newcomb unless he can realise the possibility of what I am quite sure is a fact—that we are not employing in our practical astronomy the true mean solar day as our unit of time. To assume that such is the case without proof is simply to assume the whole matter at issue. That the same angle can be represented by either $(360^\circ+n)t$ or $(360^\circ+n+\delta n)t$ is simply impossible; and that the angles $360^\circ t$ and nt can be separated, tabulated on different scales, and the results afterwards combined without error, is equally impossible. That the errors introduced by such suppositions can be neglected in the present state of astronomy is clearly a matter of proof and not of assumption.

But even if Prof. Newcomb does believe, as astronomers have hitherto believed, that we do use the true mean solar day as our unit of time in our practical astronomy, I beg of him, from the importance of the subject, to simply follow my ideas to their logical consequences, and to examine whether there are any facts inconsistent with them, and whether the results deducible from them do not afford explanations of discordances which have hitherto been otherwise inexplicable. It is only in this way that advances in science can be made.

Let

$$(1)\quad \text{R.A. of Greenwich meridian} = s = a + \left(\frac{n}{15} + 24^{\text{h}}\right)t + \sigma t^2 + \text{nutation}.$$

The epoch being 1800 Jan. 1, G.M.T., and t reckoned in units T, or so-called mean solar days whose length has hitherto been supposed to be given by the equation

$$\text{T} = \text{sidereal day}\left(\frac{360^\circ+n}{360^\circ}\right),$$

when A and n are Bessel's adopted constants we know, as a matter of fact, that Hansen's Lunar Tables represent the observations very closely from 1750 to 1864, when we determine our time t from (1) in the usual manner. If a careful examination of our method of determining mean time is made,

or of the method of determining the R.A. and N.P.D. on the meridian, for comparison with our observations, from the tabular places for assumed Greenwich mean times, which are directly given by our Lunar or Solar Tables, it will be seen that we start each day from the assumed R.A. of the meridian at "mean noon" as a *fixed point*. All the rest of the calculation is merely differential, and any resulting errors are small from the rapid rotation of the Earth upon its axis. If, therefore, we do change our unit of time we must so change it that our tabular places may correspond still to the true sidereal time at the adopted mean noon used in the construction of the tables for the determination, on the scale of time T, of the tabular mean motions. It is from the want of a clear perception of the importance of this point that the present discussion has been possible. If this condition was not necessary we could change the unit of time by putting

$$t = \left(1 + \frac{\delta n}{360° + n}\right) t',$$

as is generally assumed to be the case, and then determine t' from s from the equation

$$s = a + \left(24^{h} + \frac{n + \delta n}{15}\right) t' + \text{etc.} \ldots$$

But taking $t' = r$ an integer, we deduce from this expression

$$sn = a + (n + \delta n) r,$$

and not nr, which is its value at the *true* tabular mean noon of our adopted scale.

If, therefore, we will change our unit of time, and want to still tabulate the sidereal times at mean noon and to avoid errors, we must change the units so that nt and $360°t$ are proportionately changed. If we put

$$nt = (n + \delta n) t',$$

then

$$s = a + \left\{ \frac{(n + \delta n)}{15} t' + 24^{h} \left(1 + \frac{\delta n}{n}\right) t' \right\} + \text{etc.};$$

and we have, when $t = r = r^{\text{th}}$ noon of original scale, to which our Lunar Tables correspond,

$$sn = a + nr + \text{etc.},$$

which is the true value. But if I use, instead of the expression

$$s = a + \left(\frac{n + \delta n}{15}\right) t' + 24^{h} . t' + 24^{h} . \frac{\delta n}{n} . t' + \sigma \left(1 + \frac{\delta n}{n}\right) . t'^{2} + \text{nutation},$$

the expression

$$s = a + \left(\frac{n + \delta n}{15}\right) t' + 24^{h} . t' + \sigma t'^{2} + \text{nutation},$$

the first gives me the correct, the second the erroneous result, when t' is put equal any integer r, and the result used as the sidereal time corresponding to that of the r^{th} noon from the original epoch. I think I am fairly entitled, therefore, to ask why I must use the erroneous result, and how the tabular quantities can be expected to agree with observations when I thus clearly make a mathematical error, and how Hansen's Tables are supposed to adapt themselves to changes made in 1858 or 1864, after their construction? The whole question, therefore, turns upon this point: Can I, so long as I determine my mean times as at present, neglect the term

$$24^{\text{h}}.\frac{\delta n}{n}.t'$$

in the expression for s when the term nt has been changed into

$$(n+\delta n)t'?$$

If I can it must be proved, not assumed; and no proof has as yet been attempted. Directly I introduce the correction required to secure continuity in our measures of time, viz.

$$24^{\text{h}}.\frac{\delta n}{n}.t'$$

into the expression for the R.A. of the meridian, all Prof. Newcomb's difficulties disappear.

For if the American and English astronomers are simply neglecting the same term

$$24^{\text{h}}\frac{\delta n}{n}.t'$$

in their computations of the so-called sidereal times at mean noon, then their calculated sidereal times at mean noon will agree and be equally in error; and therefore their deduced mean times will be equally in error; and these equal errors in mean time will cause the tabular places computed for erroneous times to be equally in error, and the discordances between the tabular places and the Greenwich and American observations will, therefore, be equally large after the year 1864. There will be differences between 1858 and 1863, but these differences will be small, and will be considerably disguised by the fact that Hansen's longitudes were too small during the years 1858–1863, as shown by the Greenwich observations; and the effect of the change during these years will be simply to make the tabular places agree rather more closely with the observations.

In the following table I give the excess of Hansen's longitude over observation for the Greenwich observations for some years about the critical period 1858–1864. The value of (H−O) in longitude from the Greenwich observations was $+1''$ in 1847, and then continued about $-1''$ until 1864. The excess now is over $14''$.

The Greenwich observations for the years 1859, 1860, and 1861 have not yet been compared with Hansen's Tables, but this is now to be done by the Society. Supposing, however, as Prof. Newcomb states, that the Washington observations agree closely with the Greenwich observations, I have given some residual errors on the supposition that the Washington observations give identical results with those derived from the Greenwich observations. It will be seen that no reliable evidence either for or against my views can be thus obtained.

Year.	Error due to Change in Scale of Time between Washington and Greenwich.	Observed Value of H−O from Greenwich Observations (Longitude).	Washington Residuals corrected for Error in Time on the supposition that the Uncorrected Residuals are identical with those obtained from the Greenwich Observations.
	″	″	″
1855	0·0	−0·9	−0·9
1856	0·0	−1·0	−1·0
1857	0·0	−1·9	−1·9
1858	+0·4	−2·0	−1·6
1859	+1·1	$-x$	$+1{\cdot}1-x$
1860	+1·9	$-y$	$+1{\cdot}9-y$
1861	+2·6	$-z$	$+2{\cdot}6-z$
1862	+3·3	−2·8	+0·5
1863	+4·1	−1·6	+2·5
1864	0·0	+0·1	+0·1
1865	0·0	+1·2	+1·2
1866	0·0	+2·1	+2·1
1867	0·0	+3·5	+3·5

It is impossible to fix empirically within several years the exact period at which a regular increase of 0″·7 per annum commences, when the periodic fluctuations amount to several seconds. For instance, there is actually a decrease of H−O between the years 1875 and 1878, due to periodic and other errors; and therefore I lay no stress whatever upon the fact, but the application of these corrections would sensibly make the Washington observations appear to run away from about 1863, and Prof. Newcomb asserts that such is actually the case. The Washington lunar observations are, however, unfortunately not very numerous or well distributed over every lunation.

The correction in the years 1859, 1860, 1861 would therefore simply tend to produce a somewhat better agreement between the tabular places and the observed. If I had attempted to determine the period at which the regular increase commenced empirically I should not have fixed on the year 1864, but 1862, and should then have had to compute the residuals for 1859, 1860, 1861, before attempting to decide such a point empirically. For it is quite possible that Hansen's Tables might be in defect

instead of excess in 1860 or 1861 to a greater extent than in 1864. It was not, however, until after I had discovered the cause of the running away and found its amount approximately from an examination of the solar theory that I had these mean errors, or discordances, between observations and Hansen's Tables collected. There is, therefore, not the slightest force in Prof. Newcomb's dilemma unless he can prove, and not assume, that the term

$$24^{h}.\frac{\delta n}{n}.t'$$

is not required as a correction to

$$s = a + \left(\frac{n}{15} + 24^{h}\right)t'$$

to reduce our mean times to the scale of those adopted before 1864 in the British and before 1858 in the American *Nautical Almanacs*, when we determine our mean times from the adopted sidereal times at mean noon and refer our Right Ascensions of the stars used for clock error to mean times through the tropical year as at present. t' is here reckoned from the original epoch, 1800 Jan. 1, G.M.N.

With reference to Prof. Newcomb's statements A, B, and C, I have given not only the discordances between Hansen's Tables and the Greenwich observations when the times are uncorrected, but also the errors when the times are reduced to the standard of Carlini's Tables by the application of the proper correction,

$$24^{h}.\frac{\delta n}{n}.t',$$

for the Greenwich observations from 1864 to 1881, and for the Radcliffe observations during the years 1882 and 1883. All these results accord with the correctness of my views. The necessity for this correction dates from 1864 Jan. 1, G.M.N., for the Greenwich and Radcliffe observations. This is my answer to A and B. Of course the correction

$$24^{h}.\frac{\delta n}{n}.t'$$

must only be applied when the mean times actually adopted for the reduction of the observations have been computed with the value $n+\delta n$ instead of the value n; and it would only be whilst the times were thus computed for successive years that an agreement between Hansen's Tables and observations could be made by the method I pointed out in the *Monthly Notices* (vol. xliii. No. 7) for May 1883 by applying a decrease of $0''.747$ per annum to the mean longitudes of Hansen's Tables, the decrease being of course reckoned from the time the error first commenced till the time the error ceases to be made; and in the observations I was discussing this change was, as I clearly stated, made in 1864,

and I have actually drawn a line between the results for 1863 and 1864 in the tabular matter to show the break of continuity (see p. 337). I am, therefore, at a loss to understand Prof. Newcomb's doubt about the meaning of my *t*.

At present I have not attempted to correct Hansen's Tables at all, or touched their results in any way whatever, but merely presented the real errors of Hansen's Tables. This is my answer to question C. Prof. Newcomb's evident misconceptions about this matter are due simply to his not realising the fundamental character of the question. Two independent things are required before tabular places can be compared with observations —viz. the tabular places computed directly from the Tables, and the *mean times from observation* for which the tabular places are to be interpolated or directly computed. If your tables are wrong the results will disagree from observation; but if *you will change the scale upon which the time is measured, and not allow for the differences between your times and those adopted for the construction of the tables*, then the resulting discordances are not due to the tables at all, but to your work, *and these errors will date not from the epoch of the tables, but from the time you commenced to make those changes.* There is one point to which attention should be called. If n is Bessel's adopted value of the Sun's mean motion, and ω the angular velocity of the Earth about its axis, we have

$$\frac{\text{Tropical year}}{\text{Sidereal day}} = \frac{\omega}{n}.$$

If, therefore, we consider, as we have hitherto done, that ω is an absolute constant—a supposition which cannot be true if our unit of time is changed in the slightest degree—but that n can vary, we must have different lengths of the tropical year with every change of n, and the result to which this leads is as follows.

The tropical year for a long period to 1858 contains some definite number of sidereal seconds. In 1858 we had two tropical years differing $1^{s}\cdot46$, according as we took up the English or American *Nautical Almanac*. In 1864 the English changed their tropical year. Now I would ask, What is the meaning of such changes in estimation? They must be due to errors in our methods of measuring time; and the magnitude of the errors thus indicated is *exactly* that which is shown by the discordances between theory and observation in the case of the lunar observations, and confirmed, although of course only slightly shown, by the solar observations also. If my views are accepted there is no change of the tropical year at all. All I assume is that nothing of a physical character has changed.

The comparatively large change of unit which took place in 1864 is, however, mainly due to the artificial manner in which our mean times are computed. This method has been based on the assumption that we were, in all cases, using the true mean solar day as our unit; and we have attempted to keep the

principal term in s, $24^h \times t$, unchanged, however much we may have changed n, and therefore our unit of time, and have thus thrown the adjustment either upon the nt or upon the a, which has thus become a variable instead of a constant.

If we redetermined all our mean times, starting with the constants now adopted in Le Verrier's Solar Tables, and therefore no longer referring our tabular places to the mean noons on Bessel's scale, then the adopted unit of time would only appear to contain about $24^h\ 3^m\ 56^s{\cdot}555144$.

The effects of the term

$$24^h . \frac{\delta n}{n} . t'$$

would then be included in the new determinations of the constants of mean motion of the Sun, Moon, and planets, which would or should refer the tabular places to the new mean noons. In the present case I have treated the question entirely upon the supposition that the n and $n+\delta n$ are mere adopted angles, and assume no physical connection with the Sun's mean motion at all directly, but merely continuity in our time measures. Indirectly, however, the assumption is tacitly made in our practical work by the way in which our Right Ascensions are referred to epoch through the tropical year. I am, however, led to exactly the same results as when I discuss the question under the view that n and $n+\delta n$ both represent the Sun's real motion in longitude. In the latter case, however, the true unit of time T is seen at once to be given by an equation of condition connecting the true mean solar day $T_\circ$, the sidereal day s, T, and n; for

$$\omega = n + 360^\circ . \frac{T}{T_\circ},$$

an equation which I have proved from first principles, and which must be proved to be erroneous before I can be shown to be wrong, since if it be true we have attempted to carry on our approximations by retaining a term δn and neglecting the term

$$360^\circ \frac{\delta n}{n},$$

which is 365·25 times as large. Under this view, if we assume that we are using true mean solar times, my results simply give the differences due to the errors of this assumption in true mean solar time arising from the change of units.

If Prof. Newcomb means by A that I ought to assume that you can now recover the mean times of Carlini's scale by merely replacing Le Verrier's sidereal times at mean noons by Bessel's, with the erroneous assumption that these mean noons will thus refer to the same instants of absolute time as they would have done had Carlini's Solar Tables been continued in use since 1864, and this although *Bessel's tropical year, which differs* $1^s{\cdot}46$ *from that adopted since* 1864, *has been replaced by Le*

Verrier's to connect the sidereal times and mean times, then all I can say is, that I have done nothing, in my opinion, so absurd. To do this would simply be to assume that no change in our adopted unit of time did take place, although I have proved that in our computations of time we did replace the unit formerly in use by one which contained $0^{s}.004$ per diem more, or $1^{s}.46$ per year more. If I neglected this change I should simply get an erroneous result. The same thing has been proved by Prof. Cayley, who admits a change of unit equal to the 365th part of the change which I indicate; but who, in order to keep this change small, has to admit that he starts in 1864 *an independent mean Sun.* In such a case, of course, all our measures of time would be discontinuous after the change with those in use before the change; and to connect the two systems corrections would be required to be applied. And these corrections and his change of unit are simply those which are obtained by me combined by so changing the unit of time that no discontinuity is introduced. It appears to me that some astronomers are conducting this discussion as if we could have, compatible with accurate results, two independent and *inconsistent* measures of time—sidereal time and mean solar time. It will be found that I cannot be proved to be wrong by *simply assuming results which cannot be proved to be true except upon the assumption that we are using the true mean solar as our unit of time*, and then pointing out the fact that from these results there can be deduced consequences inconsistent with my views. I assert that we cannot, so long as we are changing from time to time our adopted values of the Sun's mean motion in longitude, measure our times from the motion of the meridian in Right Ascension in terms of the true mean solar day. And I have pointed out inconsistency after inconsistency to which the assumed truth of this has led us. But what I assume is at least a possibility, a thing within our power. I merely assume that, instead of using the time of the rotation of the Earth about its axis, or the "true sidereal day," as defined strictly by the time in which the Right Ascension of the meridian, cleared from the secular term and from the periodical terms of the nutation in R.A., increases 360°, we have preferred, as a matter of convenience, to adopt as our unit some number of sidereal seconds, which we believe will be found not to differ much from the length of the true mean solar day, whenever that interval of time shall have been accurately determined. Such a unit of time, once adopted, cannot change unless we change it. And if we change it we must allow in the usual way for the change, or we cannot escape the consequences of our neglect. The effects of changes when the unit is thus fixed are easily investigated, but the amount of the change will depend upon the way we use any adopted expression for the R.A. of the meridian

$$= \left(24^{h} + \frac{n}{15}\right) t$$

to find t. If we assume that $24^{h}.t$ is independent of changes of nt, we assume that which cannot be true. If we use the expression after the change as if $24^{h}.t$ was independent of changes in nt, we simply replace by supposition the suppressed factor

$$\frac{T}{T_0}$$

of my formula

$$s = a + \left(24^{h}\frac{T}{T_0} + \frac{n}{15}\right)t,$$

and thus really find, instead of time on scale t after the change of n to $n+\delta n$, time on scale t_1, such that

$$nt = (n+\delta n)t'.$$

I have not thought it desirable to formally discuss Prof. Newcomb's statements about the epoch 1728, since they are obviously due to some serious misconception of the question at issue.

If Prof. Newcomb will refer to p. 266 of his "Researches on the Motion of the Moon," where he has given the numbers which must be subtracted from the times indicated by astronomical observations in order to reduce them to a *uniform measure of time*, viz. for

	s		s
1625	+12	1775	−15
1650	+35	1800	−17
1675	+34	1825	− 8
1700	+25	1850	0
1725	+16	1875	+14
1750	0		

whilst for −687 the error is supposed to be -70^{m}; and if he will ponder on his own remark: "These corrections are so minute that their independent detection by existing observations is barely possible," he will perhaps realise that it is no more possible to recover *absolute epochs* after changes in the data, unless the *principle* upon which such changes are to be allowed for is settled, than it would be to detect real changes in the absolute times of the Earth's rotation, supposing such were really taking place to any sensible extent.

The Right Ascension of the meridian is definite enough; but we have in our astronomical work to connect that definite direction with some mean time t. If we introduce changes into our determinations of t, and continue these for twenty years, it is simply impossible for us to recover the exact time which corresponds to a given Right Ascension of the meridian unless we can connect the present system of mean time measures with those in use in the past without any *breaks of continuity*; and if we had two independent systems of time, sidereal time and mean solar time, it would be impossible to pass from the one to the other, after breaks of continuity had been introduced, until the proper corrections to remove these breaks of continuity had been applied. I assert that I have given the mean times which

would have been in use had Carlini's Tables been continued unaltered to the present time, and I have shown that when these necessary corrections are applied the discordances between theory and observation are sensibly removed.

I have given four independent proofs of the correctness of my views.

1. I have proved from first principles that

$$\omega = n + 360^\circ \frac{T}{T_\circ},$$

so that when we adopt a numerical value n for the Sun's mean motion in longitude, we do adopt a unit of time T in our practical astronomy just as much as when we adopt 360° for ω we adopt the sidereal day as our unit. The above equation is exact, and our expressions for sidereal times at mean noon are also exact; in fact, our time measures are exact, but expressed in terms of our unit T, whose ratio to the true mean solar day will only be exactly known when the ratio of the true mean solar day to the sidereal day is known, or when our solar theory is practically perfect.

Are these proofs of mine sound or unsound? If they are sound, then I must be right, since we have hitherto neglected the term

$$360^\circ \frac{\delta n}{n}$$

in the value of

$$\delta\omega = \delta n + 360^\circ \frac{\delta n}{n}$$

in carrying out our approximations, and have neglected the effects of the change of unit with every change of n.

The following are direct observational tests:—

2. In the following Table I show that the Right Ascensions of the stars do run away from their mean time epoch to the extent indicated by my views.

The Right Ascensions of the stars, by which our clock errors are determined, for the beginning of each year refer to the following mean times according as we employ Bessel's or Le Verrier's data: and this is the only relationship *directly established by observation between the sidereal times and mean times.*

Mean time of Sun's mean longitude being equal to 280° according to Bessel's or Le Verrier's scale of mean times.

	Bessel.		*Le Verrier.*		*LV.−B.*	
	d h m s		d h m s		m s	
1860	Dec. 31 15 13 11		Dec. 31 15 9 56		−3 15	7
1865	30 20 17 8		30 20 13 46		−3 22	7
1870	31 1 21 5		31 1 17 36		−3 29	7
1875	31 6 25 2		31 6 21 26		−3 36	8
1880	31 11 28 59		31 11 25 15		−3 44	7
1885	30 16 32 55		30 16 29 4		−3 51	

or change in 20 years = 29ˢ.

It will be seen, therefore, that instead of the change in mean time due to the change from Bessel's to Le Verrier's data being measured by the mere numerical difference of the sidereal times at mean noon according to Bessel and Le Verrier on the false assumption that no allowance is required for the change of the position of the meridian between the two mean noons, these changes are 366 times greater. In fact, the adjustment has to be made by the Sun and not by the meridian, as has hitherto been assumed. It is clear, therefore, that to find the time on *Bessel's scale* when using Le Verrier's times, as we have done in England since 1864, we must apply the corrections indicated by my theory, or our tabular places are not correctly referred to the meridian.

3. Le Verrier's Solar Tables are sufficiently good to put in evidence the same change of unit; I have shown this by a discussion of the Greenwich observations since 1864.

4. Hansen's Lunar Tables indicate most *distinctly* a similar running away to exactly the same extent; and, on account of the rapid motion of the Moon the discordance between observation and theory uncorrected for the change of mean time already amounts to more than $14''$, a quantity sufficiently large to be safely separated from personal and instrumental defects.

The Apparent Orbit of a Satellite of a Superior Planet. By J. Morrison, M.D., M.A., Ph.D., Assistant on the American Ephemeris, Washington, D.C.

The apparent orbit of the satellite of a superior planet is the projection of the real orbit on a plane perpendicular to the line joining the Earth and planet, and is, therefore, in general an ellipse. In this paper I shall assume that the elements of the orbit of the satellite are known, and proceed to develop as much of the subject as is necessary for the computation of an ephemeris.

Let

α, δ, Δ be the polar coordinates of the planet as seen from the Earth;

$\alpha', \delta', \Delta'$ the polar coordinates of the satellite as seen from the Earth;

a, d, r the polar coordinates of the satellite as seen from the planet,

referred to the equator and equinox of the epoch for which the ephemeris is to be computed.

The general expressions for the rectangular coordinates in terms of polar coordinates are

$$x = \Delta \cos \delta \cos \alpha$$
$$y = \Delta \cos \delta \sin \alpha$$
$$z = \Delta \sin \delta.$$

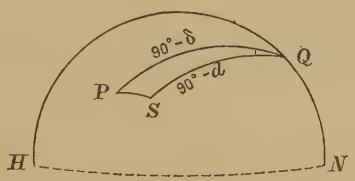
Therefore by equating the rectangular coordinates we have

$$\begin{aligned} \Delta' \cos\delta' \cos\alpha' &= \Delta \cos\delta \cos\alpha + r \cos d \cos a \\ \Delta' \cos\delta' \sin\alpha' &= \Delta \cos\delta \sin\alpha + r \cos d \sin a \qquad \ldots \quad (1) \\ \Delta' \sin\delta' &= \Delta \sin\delta \quad + r \sin d. \end{aligned}$$

Multiplying the first of this group by cos α, the second by sin α, and adding the results, then multiplying the first by sin α, the second by cos α, and subtracting, we have

$$\begin{aligned} \Delta' \cos\delta' \cos(\alpha'-\alpha) &= \Delta \cos\delta + r \cos d \cos(a-\alpha) \\ \Delta' \cos\delta' \sin(\alpha'-\alpha) &= r \cos d \sin(a-\alpha) \qquad \ldots \quad \ldots \quad (2) \\ \Delta' \sin\delta' &= \Delta \sin\delta + r \sin d. \end{aligned}$$

We have now to consider the spherical triangle formed by the north pole of the equator, the planet, and the satellite; thus let Q be the north pole, P the planet, and S the satellite. In the spherical triangle Q P S we have



$$PQ = 90° - \delta,\ QS = 90° - \delta';$$

and

$$PQS = \alpha' - \alpha.$$

Put

PS = s, the apparent angular distance of the satellite from the planet as seen from the Earth, and

QPS = p, the angle of position of the satellite reckoned from the north toward the east and from 0° to 360°.

By the fundamental formulæ of spherical trigonometry we have

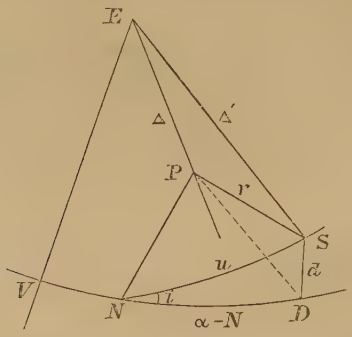
$$\begin{aligned} \sin s \sin p &= \cos\delta' \sin(\alpha'-\alpha) \\ \sin s \cos p &= \cos\delta \sin\delta' - \sin\delta \cos\delta' \cos(\alpha'-\alpha) \qquad \ldots \quad (3) \\ \cos s &= \sin\delta \sin\delta' + \cos\delta \cos\delta' \cos(\alpha'-\alpha) \end{aligned}$$

Multiplying by Δ' and substituting from (2) this group becomes

$$\begin{aligned} \Delta' \sin s \sin p &= r \cos d \sin(a-\alpha) \\ \Delta' \sin s \cos p &= r \{\cos\delta \sin d - \sin\delta \cos d \cos(a-\alpha)\} \qquad \ldots \quad (4) \\ \Delta' \cos s &= \Delta + r \{\sin\delta \sin d + \cos\delta \cos d \cos(a-\alpha)\}, \end{aligned}$$

from which we must eliminate a, d, and Δ'.

Let V D represent an arc of the equator, N S an arc of the orbit of the satellite, E the Earth, P the planet, and S the satellite; draw the arc S D perpendicular to V D, and let E V be the direction of the vernal equinox. Then N will be the



position of the ascending node of the orbit of the satellite on the equator; V D and S D the planetocentric Right Ascension and Declination of the satellite, which we have denoted by α and d respectively.

Put

VN = N, the right ascension of the ascending node;

SND = i, the inclination of the orbit to the plane of the equator;

NS = u, the angular distance of the satellite from the ascending node;

then

ND = $\alpha - N$.

From the right-angled spherical triangle N S D, whose centre is P, we have

$$\begin{aligned} \cos d \cos(\alpha - N) &= \cos u \\ \cos d \sin(\alpha - N) &= \sin u \cos i \quad \ldots \quad \ldots \quad \ldots \quad (5) \\ \sin d &= \sin u \sin i. \end{aligned}$$

Multiplying the first of this group by cos ($\alpha - N$), the second by sin ($\alpha - N$), and adding the results; also multiplying the first by sin ($\alpha - N$) and the second by cos ($\alpha - N$), and subtracting, we have

$$\begin{aligned} \cos d \cos(\alpha - a) &= \sin u \cos i \sin(\alpha - N) + \cos u \cos(\alpha - N) \\ \cos d \sin(\alpha - a) &= \sin u \cos i \cos(\alpha - N) - \cos u \sin(\alpha - N) \\ \sin d &= \sin u \sin i, \end{aligned}$$

which substituted in (4) give, after simple reductions—

$$\begin{aligned} \Delta' \sin s \sin p &= r \{\sin u \cos i \cos(\alpha - N) - \cos u \sin(\alpha - N)\} \\ \Delta' \sin s \cos p &= r \{\sin u [\cos \delta \sin i - \sin \delta \cos i \sin(\alpha - N)] \\ &\qquad - \sin \delta \cos u \cos(\alpha - N)\} \\ \Delta' \cos s &= \Delta + r \{\sin u [\sin \delta \sin i + \cos \delta \cos i \sin(\alpha - N)] \\ &\qquad + \cos \delta \cos u \cos(\alpha - N)\}, \end{aligned}$$

from which Δ' must be eliminated.

In the plane triangle E P S formed by joining the Earth, planet, and satellite, let σ denote the supplement of the angle E P S; then from the diagram (fig. 2) we have

$$\Delta' \sin s = r \sin \sigma$$

and

$$\Delta' \cos s = r \cos \sigma + \Delta \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

which being substituted in the equations of the last group, give after reduction

$$\begin{aligned} \sin \sigma \quad \text{n } p &= \sin u \cos i \cos (\alpha - \text{N}) - \cos u \sin (\alpha - \text{N}) \\ \sin \sigma \cos p &= \sin u \, [\cos \delta \sin i - \sin \delta \cos i \sin (\alpha - \text{N})] \\ &\qquad - \cos u \sin \delta \cos (\alpha - \text{N}) \quad \ldots \quad (7) \\ \cos \sigma &= \sin u \, [\sin \delta \sin i + \cos \delta \cos i \sin (\alpha - \text{N})] \\ &\qquad + \cos u \cos \delta \cos (\alpha - \text{N}). \end{aligned}$$

These equations are perfectly rigorous, and determine σ and p without ambiguity.

As the angle σ cannot be compared directly with observation, it will be more convenient to express σ in terms of s; thus from (6) we have

$$\tan s = \frac{r \sin \sigma}{r \cos \sigma + \Delta} \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

and since s is always very small, we may substitute the arc for the tangent and develope the second member into a series. Hence we have

$$s = \frac{r}{\Delta} \sin \sigma - \tfrac{1}{2} \frac{r^2}{\Delta^2} \sin 2\sigma + \tfrac{1}{3} \frac{r^3}{\Delta^3} \sin 3\sigma - \text{ etc.} \quad \ldots \quad (9)$$

Now r is always very small compared with Δ, and therefore the terms containing the square and higher powers of $\frac{r}{\Delta}$ may be neglected without sensibly affecting the result. We shall then have

$$s'' = \frac{r}{\Delta \sin 1''} \sin \sigma \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

and if a'' be the value of $\frac{r}{\sin 1''}$ at distance unit $\frac{a''}{\Delta}$ will be its value at distance Δ; therefore the last equation may be written

$$s'' = \frac{a''}{\Delta} \sin \sigma,$$

whence

$$\sin \sigma = \frac{\Delta}{a''} . s'' \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

which, substituted in the first two equations of (7), gives

$$s'' \sin p = \frac{a''}{\Delta}\left\{\sin u \cos i \cos(\alpha - N) - \cos u \sin(\alpha - N)\right\} \quad \ldots \quad \ldots \quad (12)$$

$$s'' \cos p = \frac{a''}{\Delta}\left\{\sin u\,[\cos\delta \sin i - \sin\delta \cos i \sin(\alpha - N)] - \cos u \sin\delta \cos(\alpha - N)\right\}$$

The error committed in s'' by omitting the second term of (9) is

$$\tfrac{1}{2}\,\frac{r^2}{\Delta^2}\,\frac{\sin 2\sigma}{\sin 1''},$$

which is a maximum when

$$\sigma = 45° \text{ or } 135°.$$

Put

$$\tfrac{1}{2}\,\frac{r^2}{\Delta^2}\,\frac{1}{\sin 1''} = \eta;$$

then

$$\frac{r}{\Delta} = \sqrt{\eta \sin 2''};$$

but $\frac{r}{\Delta}$ is the tangent of the angle of greatest elongation as seen from the earth; hence, putting

$$\frac{r}{\Delta} = \tan\epsilon,$$

we have

$$\eta = \frac{\tan^2\epsilon}{\sin 2''},$$

when

$\epsilon = 0^\circ\ 50'',$	$\eta = 0''{\cdot}006$
$\epsilon = 1\ 50,$	$\eta = 0{\cdot}029$
$\epsilon = 3\ 35,$	$\eta = 0{\cdot}112$
$\epsilon = 5\ 46,$	$\eta = 0{\cdot}290$
$\epsilon = 9\ 45,$	$\eta = 0{\cdot}829$

Thus we see that for the satellites of *Mars*, *Uranus*, *Neptune*, the three inner satellites of *Jupiter*, and the five inner satellites of *Saturn* the error committed in s'' is less than $0''{\cdot}3$, and for *Callisto*, the fourth satellite of *Jupiter*, which has the largest angle of elongation, the error can never exceed $0''{\cdot}83$. In order to adopt (7) and (12) to logarithmic computation put

$$\begin{aligned} \sin f \cos \mathrm{F} &= \cos i \cos(\alpha - N) \\ \sin f \sin \mathrm{F} &= -\sin(\alpha - N) \quad \ldots \quad \ldots \quad \ldots \quad (13) \\ \cos f &= -\sin i \cos(\alpha - N) \end{aligned}$$

$$\begin{aligned} \sin g \cos \mathrm{G} &= \cos\delta \sin i - \sin\delta \cos i \sin(\alpha - N) \\ \sin g \sin \mathrm{G} &= -\sin\delta \cos(\alpha - N) \quad \ldots \quad \ldots \quad (14) \\ \cos g &= \cos\delta \cos i + \sin\delta \sin i \sin(\alpha - N) \end{aligned}$$

$$\begin{aligned} \sin h \cos \mathrm{H} &= \sin \delta \sin i + \cos \delta \cos i \sin (\alpha - \mathrm{N}) \\ \sin h \sin \mathrm{H} &= \cos \delta \cos (\alpha - \mathrm{N}) \quad \ldots \quad \ldots \quad \ldots \quad (15) \\ \cos h &= \sin \delta \cos i - \cos \delta \sin i \sin (\alpha - \mathrm{N}). \end{aligned}$$

Hence (7) and (12) become

$$\begin{aligned} \sin \sigma \sin p &= \sin f \sin (\mathrm{F} + u) \\ \sin \sigma \cos p &= \sin g \sin (\mathrm{G} + u) \quad \ldots \quad \ldots \quad \ldots \quad (16) \\ \cos \sigma &= \sin h \sin (\mathrm{H} + u) \end{aligned}$$

and

$$\begin{aligned} s'' \sin p &= \frac{a''}{\Delta} \sin f \sin (\mathrm{F} + u) \\ s'' \cos p &= \frac{a''}{\Delta} \sin g \sin (\mathrm{G} + u) \quad \ldots \quad \ldots \quad (17) \end{aligned}$$

The last group, which is known as Bessel's formulæ, gives s'' and p for any given date, and also enables us to compare the computed place with that obtained by observation. When s and p are to be computed for a series of dates it will be most convenient to compute the values of the auxiliary angles, f, F, g, and G, for the mean noon of several consecutive days, and then interpolate for any intermediate date. The auxiliaries h and H will not be required unless we also want the value of σ.

The following relations among the auxiliary angles may be here noticed, since they will be required presently in the reduction of the differential equations.

By combining the values of $\sin f \sin \mathrm{F}$, $\sin g \cos \mathrm{G}$, $\sin f \cos \mathrm{F}$, and $\sin g \sin \mathrm{G}$, we have

$$\begin{aligned} \sin f \sin g \sin \mathrm{F} \cos \mathrm{G} &= \sin^2 (\alpha - \mathrm{N}) \cos i \sin \delta - \sin (\alpha - \mathrm{N}) \sin i \cos \delta \\ \sin f \sin g \cos \mathrm{F} \sin \mathrm{G} &= -\cos^2 (\alpha - \mathrm{N}) \cos i \sin \delta; \end{aligned}$$

therefore by subtraction we have

$$\begin{aligned} \sin f \sin g \sin (\mathrm{F} - \mathrm{G}) &= \cos i \sin \delta - \sin (\alpha - \mathrm{N}) \sin i \cos \delta \\ &= \cos h \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (18) \end{aligned}$$

In a similar manner we find

$$\begin{aligned} \sin g \sin h \sin (\mathrm{G} - \mathrm{H}) &= \cos f \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (19) \\ \sin f \sin h \sin (\mathrm{H} - \mathrm{F}) &= \cos g \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (20) \end{aligned}$$

Again, multiplying the first and second of (15) by $\cos u$ and $\sin u$ respectively, we have, after obvious reductions—

$$\begin{aligned} &\sin h \cos (\mathrm{H} + u) \\ &\quad = [\sin i \sin \delta + \cos i \cos \delta \sin (\alpha - \mathrm{N})] \cos u - \cos \delta \cos (\alpha - \mathrm{N}) \sin u; \end{aligned}$$

also, multiplying the second members of the first and second of (16) by the third of (14) and (13) respectively, we have, by the aid of (7)—

$$\begin{aligned} \cos f \sin g \sin (\mathrm{G} + u) - \sin f \cos g \sin (\mathrm{F} + u) &= [\sin i \sin \delta \\ &+ \cos i \cos \delta \sin (\alpha - \mathrm{N})] \cos u - \cos \delta \cos (\alpha - \mathrm{N}) \sin u. \end{aligned}$$

Comparing this with the preceding equation, we have the following relation :

$$\sin h \cos (H+u) = \cos f \sin g \sin (G+u) - \sin f \cos g \sin (F+u) \quad (21)$$

To Correct the Elements.

When the computed places do not agree with those obtained by observation it will be necessary to correct the elements by forming equations of condition, which will be of the form

$$\Delta p = \frac{dp}{du}\Delta u + \frac{dp}{dN}\Delta N + \frac{dp}{di}\Delta i$$

$$\Delta s = \frac{ds}{du}\Delta u + \frac{ds}{dN}\Delta N + \frac{ds}{di}\Delta i + \frac{ds}{da}\Delta a,$$

where we consider for the present the elements of a circular orbit.

Differentiating the first of (7) we get (α and δ being constants)

$$\begin{aligned}\sin\sigma\cos p dp + \cos\sigma\sin p d\sigma = {} & [\sin(\alpha-N)\sin u + \cos(\alpha-N)\cos i\cos u]du \\ & + [\cos(\alpha-N)\cos u + \sin(\alpha-N)\cos i\sin u]dN \\ & - \cos(\alpha-N)\sin i\sin u di \\ = {} & \sin f\cos(F+u)du \\ & + [\cos(\alpha-N)\cos u \\ & \qquad + \sin(\alpha-N)\cos i\sin u]dN \quad (a) \\ & + \cos f\sin u di\end{aligned}$$

Differentiating the second of (7) gives

$$\begin{aligned}-\sin\sigma\sin p dp + \cos\sigma\cos p d\sigma = {} & [\cos(\alpha-N)\sin\delta\sin u - \sin(\alpha-N) \\ & \qquad \cos i\sin\delta\cos u + \sin i\cos\delta\cos u]du \\ & + [\cos(\alpha-N)\cos i\sin\delta\sin u \\ & \qquad - \sin(\alpha-N)\sin\delta\cos u]dN \\ & + [\sin(\alpha-N)\sin i\sin\delta\sin u \\ & \qquad + \cos i\cos\delta\sin u]di \\ = {} & \sin g\cos(G+u)du \\ & + \sin\delta\sin f\sin(F+u)dN \quad \ldots \quad (b) \\ & + \cos g\sin u di\end{aligned}$$

Multiplying (a) by the second of (16), and (b) by the first of (16), and subtracting the latter result from the former, we have, after reducing by the aid of the relations (18–21)—

$$\begin{aligned}\sin^2\sigma\, dp = {} & -\cos h du \\ & + [\sin i\sin h\sin(H+u) - \sin\delta]dN \quad \ldots \quad (22) \\ & + \sin h\cos(H+u)\sin u di.\end{aligned}$$

Differentiating the third of (7), we find after similar reductions

$$\begin{aligned}\sin\sigma d\sigma = &-\sin h\cos(\Pi+u)du\\ &+\cos\delta\sin f\sin(\mathrm{F}+u)d\mathrm{N}\\ &-\cos h\sin u di.\end{aligned}\quad\ldots\quad\ldots\quad(23)$$

From the plane triangle E P S (fig. 2) we have

$$\Delta = \Delta\cos s + r\cos(\sigma-s)$$

and

$$\sin s = \frac{r}{\Delta}\sin(\sigma-s).$$

Differentiating the latter and reducing by the former we have

$$\frac{\sin\sigma}{\sin s}ds = \cos(\sigma-s)d\sigma+\sin(\sigma-s)\frac{dr}{r}\quad\ldots\quad\ldots\quad(24)$$

whence

$$d\sigma = \frac{\sin\sigma}{\sin s\cos(\sigma-s)}ds-\tan(\sigma-s)\frac{da}{a},$$

where we write a for r in the circular orbit.

This being substituted in (23) gives, after putting for brevity

$$\sin s\cos(\sigma-s) = n,$$

$$\begin{aligned}\sin^2\sigma ds = &-n\sin h\cos(\Pi+u)du\\ &+n\cos\delta\sin f\sin(\mathrm{F}+u)d\mathrm{N}\\ &-n\cos h\sin u di\\ &+\frac{n}{a}\sin\sigma\tan(\sigma-s)da\end{aligned}\quad\ldots\quad\ldots\quad(25)$$

Therefore the partial differential coefficients of h and s with respect to the elements of the circular orbit are

$$\begin{aligned}\sin^2\sigma\frac{dp}{du} &= -\cos h\\ \sin^2\sigma\frac{dp}{d\mathrm{N}} &= \sin i\sin h\sin(\Pi+u)\sin u-\sin\delta\\ \sin^2\sigma\frac{dp}{di} &= \sin h\cos(\Pi+u)\sin u,\end{aligned}\quad\ldots\quad(26)$$

and

$$\begin{aligned}\sin^2\sigma\frac{ds}{du} &= -n\sin h\cos(\Pi+u)\\ \sin^2\sigma\frac{ds}{d\mathrm{N}} &= n\cos\delta\sin f\sin(\mathrm{F}+u)\\ \sin^2\sigma\frac{ds}{di} &= -n\cos h\sin u\\ \sin^2\sigma\frac{ds}{da} &= \frac{n}{a}\sin\sigma\tan(\sigma-s),\end{aligned}\quad\ldots\quad\ldots\quad(27)$$

where s and a are to be expressed in seconds of arc at distance Δ.

If, however, we take the eccentricity of the orbit into consideration, we must express du and da or dr in terms of the remaining elements of the elliptic orbit. Let

m_o denote the mean anomaly at any given epoch T_o,

μ the mean motion in a solar day,

ϕ the angle of eccentricity such that $\sin\phi = e$,

E the eccentric anomaly,

v the true anomaly,

w the angular distance from the ascending node to the pericentre,

and

m the mean anomaly for any subsequent date t;

then by the theory of elliptic motion we have

$$\begin{aligned} m &= m_o + (t - T_o)\mu \\ &= E - \sin\phi \sin E \end{aligned}$$

$$\begin{aligned} E = {} & m + (\sin\phi - \tfrac{1}{8}\sin^3\phi + \ldots)\sin m \\ & + \tfrac{1}{2}\sin^2\phi \sin 2m - \ldots\ldots \end{aligned}$$

$$\begin{aligned} \cos E = {} & -\tfrac{1}{2}\sin\phi + (1 - \tfrac{3}{8}\sin^2\phi + \ldots)\cos m \\ & + \tfrac{1}{2}\sin\phi\cos 2m - \ldots\ldots \end{aligned}$$

$$\begin{aligned} r &= a(1 - \sin\phi\cos E) \\ &= a(1 - \sin\phi\cos m + \tfrac{1}{2}\sin^2\phi - \ldots) \quad \ldots \quad (28) \end{aligned}$$

$$v = m + 2\sin\phi\sin m + \tfrac{5}{4}\sin^2\phi\sin 2m + \ldots$$

and also

$$\begin{aligned} u &= w + v \\ &= w + m + 2\sin\phi\sin m + \ldots\ldots \quad \ldots \quad (29) \end{aligned}$$

Since the eccentricity of the orbits of all the satellites of the superior planets is not only small but also very uncertain, we may neglect the square and higher powers of $\sin\phi$ without sensibly affecting the accuracy of our results.

Differentiating (29) gives

$$du = dw + dm + 2\sin\phi\cos m dm + 2\sin m\cos\phi d\phi;$$

and expressing the radius vector of the satellite in terms of the semi-major axis (a) of its orbit by putting

$$\frac{r}{a} = \rho,$$

we shall have by differentiating this last equation

$$\frac{dr}{r} = \frac{da}{a} + \frac{d\rho}{\rho},$$

where r and a are expressed in the same unit as Δ. Therefore from (28) we have

$$d\rho = d\left(\frac{r}{a}\right) = \sin\phi \sin m dm - \cos m \cos\phi d\phi;$$

hence

$$\frac{dr}{r} = \frac{da}{a} + \frac{\sin\phi \sin m}{\rho} dm - \frac{\cos m \cos\phi}{\rho} d\phi.$$

Therefore by substituting these values of du and $\frac{dr}{r}$ in (22), (24), and (25) we have for the partial differential coefficients of p and s with respect to the elements of the elliptic orbit the following results:—

$$\sin^2\sigma \frac{dp}{dw} = -\cos h$$

$$\sin^2\sigma \frac{dp}{dm} = -(1+2\sin\phi\cos m)\cos h$$

$$\sin^2\sigma \frac{dp}{d\phi} = -2\sin m\cos\phi\cos h \quad \ldots \quad \ldots \quad \ldots \quad (30)$$

$$\sin^2\sigma \frac{dp}{d\mathrm{N}} = \sin i \sin h \sin(\mathrm{H}+u)\sin u - \sin\delta$$

$$\sin^2\sigma \frac{dp}{di} = \sin h \cos(\mathrm{H}+u)\sin u$$

and

$$\sin^2\sigma \frac{ds}{dw} = -n\sin h\cos(\mathrm{H}+u)$$

$$\sin^2\sigma \frac{ds}{dm} = -(1+2\sin\phi\cos m)n\sin h\cos(\mathrm{H}+u) + \frac{n}{\rho}\sin\sigma\tan(\sigma-s)\sin\phi\sin m$$

$$\sin^2\sigma \frac{ds}{d\phi} = -[2n\sin m\sin h\cos(\mathrm{H}+u) + \frac{n}{\rho}\sin\sigma\tan(\sigma-s)\cos m]\cos\phi$$

$$\sin^2\sigma \frac{ds}{d\mathrm{N}} = n\cos\delta\sin f\sin(\mathrm{F}+u) \quad \ldots \quad \ldots \quad \ldots \quad (31)$$

$$\sin^2\sigma \frac{ds}{di} = -n\cos h\sin u$$

$$\sin^2\sigma \frac{ds}{da} = \frac{n}{a}\sin\sigma\tan(\sigma-s),$$

in which a is to be expressed in seconds of arc at distance Δ.

To Compute the Times of greatest Eastern and Western Elongation.

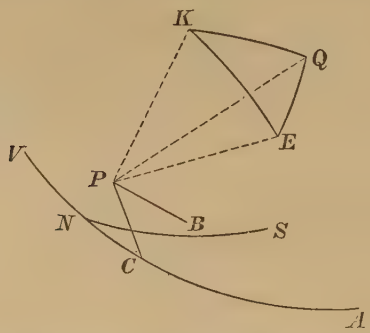
Let us conceive three planes to be passed as follows:—

The first through the centre of the planet and the poles of the equator and the orbit of the satellite;

The second through the centre of the planet, the pole of the orbit of the satellite, and the centre of the Earth;

The third through the centre of the planet, the pole of the equator, and the centre of the Earth.

These three planes will determine on the celestial sphere a spherical triangle having its centre at the centre of the planet.



Let V A represent an arc of the equator, N S an arc of the orbit of the satellite, Q the pole of the equator, K the pole of the orbit of the satellite, N the position of the ascending node, P the planet, E the Earth.

Join P E, P K, P Q, and in the plane K P E draw P B perpendicular to K P, and P C perpendicular to P E; then P B will lie in the plane of the orbit, and P C in the plane of the apparent orbit, which is the projection of the real orbit on the plane passing through P, and perpendicular to the line P E joining the Earth and planet.

Put

θ = the angle EPB,

q = the angle of position of the minor axis of the apparent orbit,

$p_0 = q \pm 90°$, the angle of position of the major axis,

v_0 = the angular distance of the end of the major axis from the ascending node measured on the orbit of the satellite,

a and b = the semi major and minor axes respectively of the apparent orbit;

then in the spherical triangle E K Q we shall have

KQ = i, the inclination of the orbit of the satellite,

EQ = $90° + \delta$, the polar distance of the Earth as seen from the planet, and

EQK = the difference between the Right Ascension of the Earth and the Right Ascension of the pole of the orbit of the satellite

$= (\alpha - 180°) - (N - 90°)$

$= \alpha - N - 90°$;

hence the remaining three parts of the triangle can be found.

Since the planes P K E, P K Q pass through the pole of the orbit, they are perpendicular to its plane, and the former intersects the orbit in those points which, as seen from the Earth, are nearest the planet, or, in other words, the intersection of the plane P K E with the apparent orbit, determining the minor axis, while the plane P K Q, being perpendicular to the planes of the equator and of the orbit of the satellite, intersects the latter at a point 90° distant from the node. The angle E K Q, being the inclination of these two planes, is measured by the arc of the orbit intercepted between these two points of intersection.

The side K Q will intersect the orbit in a point whose position is

$$90° + N;$$

and if we denote the position of the intersection of the side K E with the orbit by β we shall have

$$\text{angle EKQ} = 90° + N - \beta.$$

But since the plane P K E intersects the apparent orbit in its minor axis, the position of that point of the orbit at which the satellite, as seen from the Earth, will be at its greatest eastern elongation will be

$$\beta - 90°,$$

and therefore the distance of this point from the node will be

$$\beta - 90° - N.$$

Hence we have

$$u_0 = \beta - 90° - N.$$

But we have shown that

$$\text{angle EKQ} = -\beta + 90° + N;$$

therefore the angle E K Q $= -u_0$.

The angle K E Q is the angle of position of the minor axis of the apparent orbit; hence

$$\text{angle KEQ} = q = p_0 - 90°.$$

The side K E is the angular distance of the Earth from the pole of the orbit, as seen from the planet, and is equal to the angle B P C; therefore we have

$$\text{KE} = 90° - \theta,$$

where θ is always to be taken between $+90°$ and $-90°$. When θ is *positive* the Earth will be on the *north* side of the plane of the orbit and the position angles will *increase*, and when θ is *negative* the Earth will be on the *south* side and the position angles will *decrease*.

The fundamental formulæ of spherical trigonometry being applied to the triangle E K Q, we have

$$\begin{aligned}
\cos EK &= \cos KQ \cos EQ + \sin KQ \sin EQ \cos KQE \\
\sin EK \cos EKQ &= \sin KQ \cos EQ - \cos KQ \sin EQ \cos KQE \\
\sin EK \sin EKQ &= \sin EQ \sin KQE \\
\sin EK \cos KEQ &= \sin EQ \cos KQ - \cos EQ \sin KQ \cos KQE \\
\sin EK \sin KEQ &= \sin KQ \sin KQE.
\end{aligned}$$

But we have shown that

$$\begin{aligned}
KQ &= i, \\
EQ &= 90^\circ + \delta, \\
KQE &= \alpha - N - 90^\circ, \text{ which are known;}
\end{aligned}$$

and

$$\begin{aligned}
EK &= 90^\circ - \theta, \\
EKQ &= -u_\circ, \\
KEQ &= p_\circ - 90^\circ, \text{ which are to be found.}
\end{aligned}$$

Substituting these values of the sides and angles in the preceding formulæ, we have

$$\begin{aligned}
\sin\theta &= -\cos i \sin\delta + \sin i \cos\delta \sin(\alpha - N) \\
\cos\theta \cos u_\circ &= -\sin i \sin\delta - \cos i \cos\delta \sin(\alpha - N) \\
\cos\theta \sin u_\circ &= \cos\delta \cos(\alpha - N) \qquad \ldots \qquad (32) \\
\cos\theta \sin p_\circ &= \cos i \cos\delta + \sin i \sin\delta \sin(\alpha - N) \\
\cos\theta \cos p_\circ &= \sin i \cos(\alpha - N)
\end{aligned}$$

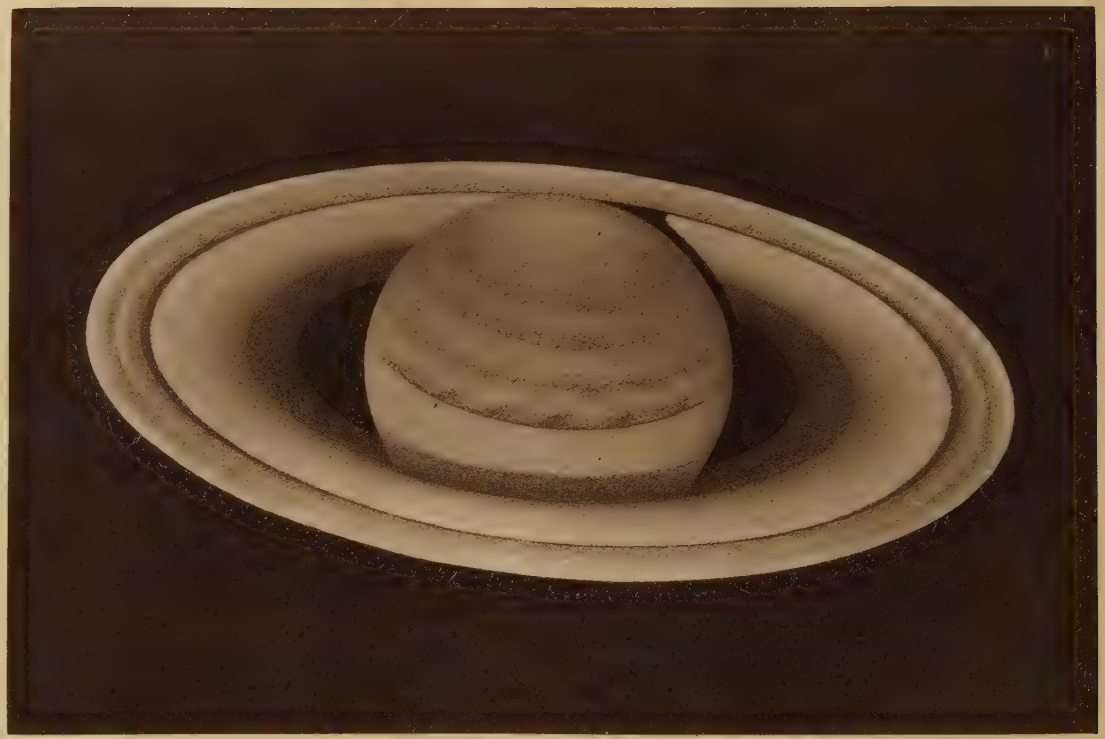
for the determination of θ, $u_\circ$, and $p_\circ$. The agreement of the two values of $\cos\theta$ and the one value of $\sin\theta$ will serve to check the accuracy of the computation. It now remains to transform these equations for logarithmic calculation.

If we put

$$\begin{aligned}
c \sin C &= \cos i \\
c \cos C &= \sin i \sin(\alpha - N), \\
\\
c' \sin C' &= \sin i \\
c' \cos C' &= -\cos i \sin(\alpha - N),
\end{aligned}$$

and remembering that for the greatest western elongation $-u_\circ$ becomes $-u_\circ + 180^\circ$, and $p_\circ$ becomes $p_\circ + 180^\circ$, we shall have

$$\begin{aligned}
\sin\theta &= c \cos(C + \delta) \\
\cos\theta \cos u_\circ &= \pm c' \cos(C' + \delta) \\
\cos\theta \sin u_\circ &= \pm \cos\delta \cos(\alpha - N) \qquad \ldots \qquad \ldots \qquad (33) \\
\cos\theta \sin p_\circ &= \pm c \sin(C + \delta) \\
\cos\theta \cos p_\circ &= \pm \sin i \cos(\alpha - N),
\end{aligned}$$



Henry Pratt, del. *Woodburytype.*

SATURN.

1884, *February* 11.

in which the upper sign is to be used for the greatest eastern, and the lower sign for the greatest western elongation.

Now if u denote the distance of the satellite from the ascending node at the date T, and if t denote the time of greatest elongation, we shall have

$$(t-\mathrm{T})\,\mu = u_0 - u,$$

whence

$$t = \mathrm{T} + \frac{u_0 - u}{\mu} \quad \ldots \quad \ldots \quad \ldots \quad (34)$$

We also have

$$b = a \cos \mathrm{KE}$$
$$= a \sin \theta \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (35)$$

which determines the semi-minor axis of the apparent orbit.

The Physical Features of Saturn, 1884. By Henry Pratt.

The coincidence of a suitable position of *Saturn* with the occurrence of several nights when definition of a superlative character favoured me with opportunities for a minute examination of the planet, is my chief reason for offering to the Society the results of many observations. The occasions referred to were chiefly in February; some occurred in January and March; but the most notable occasion was Feb. 11, when I re-observed every detail I had previously seen.

The drawing which I then made to the scale of ·15 inch $=1''$ was based upon micrometer measures, and a slightly reduced reproduction of it accompanies this paper.

The telescope used was my clock-driven Newtonian of 8·15 aperture. Its speculum, by Mr. With, gives remarkably small star disks, and in conjunction with that was used a reflecting prism of equally perfect form. The powers used have been 370, 450, and 500, chiefly the two last. All colour observations were made by Browning's achromatic power of 450.

Of the Globe.

The inclination of its axis towards the Earth on Feb. 11 was sufficient to render visible more than 20° of latitude beyond the south pole, bringing the parallel of 70° south latitude just within the southern limb.

The southern limb just covered Cassini's division.

The margin of the apparent disk of the globe generally appeared slightly deeper in tone than the more central portion, but the following limb was decidedly but delicately shaded, showing the slight approach of phase.

Positions of the Dark Belts.

Changes doubtless occur in the form and arrangement of the belts on the globe, but from a constant and close examination of them through all the various aspects which the planet presents to us, I am forced to conclude that such real changes are by no means either frequent or rapid. For, whenever a slight alteration either in the form or position of the belts has been supposed to have occurred, I have afterwards found them resolved into their usual appearance by a more careful observation assisted by better definition.

Throughout the period during which the north pole was last turned towards the Earth, I observed the same regular arrangement of the belts on the northern hemisphere as was represented by the photograph of my drawing of 1874, Sept. 14, which accompanied my previous note "On the Belts of Saturn."

Throughout the period which has passed since the planet's southern pole last began to turn towards the Earth, I have constantly observed a similar arrangement of the belts on the southern hemisphere, but in reversed order to that presented by those on the northern one. And about the time when the ring was edgewise towards the Earth, the two symmetrical systems of belts were observed to occupy similar positions, so far as they could be traced north and south of the equator.

For the sake of future comparison I have designated the dark belts by the letters α, &c., commencing with the one nearest to the equator, and prefixing the minus sign to those in the southern hemisphere.

The equatorial zone has always appeared to me of a uniform tint without any markings whatever, and to extend for an equal distance on each side of the equator, its breadth being about 20° of Saturnian latitude. It is by far the most brilliant portion of the globe, and of a cream colour.

$-\alpha$. This belt, the first south of the equator, is the most interesting of all from its composite character. The position of its northern edge (as derived from micrometer measures) is 10° south latitude. Its general tint is reddish-brown, but for about one-fourth of its total breadth on its northern margin its tone is so intensified as to become a narrow dark line. The remainder of its breadth on the southern side is softened and modulated in tone, and often exhibits darker wispy knots. On Feb. 11 I counted six of such markings, three of them being quite easily seen.

$-\beta$, the next dark belt towards the south, is of the same tint as the former one, but much lighter in general tone, and is also mottled, but in a much more indistinct manner. This is especially the case on its northern margin, as to some extent it is a reversed counterpart of $-\alpha$, but, unlike the latter, it includes no darker markings.

$-\gamma$ is a still fainter belt lying further south, its northern edge being usually the best defined.

$-\delta$, the fourth and last belt, is wider than $-\gamma$, and more difficult to see, and its margins are less defined. Both the last-mentioned belts are so faint as to be always invisible unless our atmosphere is in a very transparent condition. On favourable occasions I always see them, and, as far as it is possible to determine, their general colour is the same as that of the other belts.

Between each of the four belts above mentioned, as well as south of $-\delta$, are *zones* of lighter tone, and of a pale brownish-yellow colour, each more or less softly blended with the belts, and of a darker tone than the brilliant equatorial zone.

The polar region, lying entirely within view, is without any defined boundary, being blended very softly with the zone on its north, with which it is contrasted by its darker tone; while on its farther margin it is oppositely contrasted with the still darker southern limb. Its colour appears bluish-gray.

$+\alpha$. On each side of the globe, the ends of this belt are just visible above the margin of the ring C, with which it is easily confused unless definition is of a fine order. It thus shows that the extent of the equatorial zone at the present time is about the same on the northern as it is on the southern side of the planet's equator.

Of the Rings.

A. The marking upon this ring, which is usually termed Encke's division, was very easily seen during February and March, sometimes being very unusually distinct, as it also appeared on some occasions in 1883. But it has never to my vision seemed like a simple sharply-defined line on an even flat tint, as sometimes it has been represented. It has always appeared to me very narrow and gray, with the character of a delicately softened linear shading, varying in depth of tone from time to time. I consider it to be not a complete division, and that it does not always occupy the same position on the ring. The brightness of the outer zone of A begins to diminish about its middle, and the depth of its tone increases until it meets Encke's division. And it is remarkable that on the inner zone is a repetition of the same phenomenon until it meets Cassini's division. These shadings, or whatever else they may be termed, are not unworthy of note. On 1881, Nov. 22, Encke's division was seen just within the middle of A, but the striated appearance of the shading on its outside during moments of best definition was such as to indicate the high probability of the existence of more than one marking of the nature of Encke's division; and on some nights in last January it appeared just outside of the middle of A, while on others I concluded (after much careful attention) that it was just outside of the middle on one ansa and just

within it on the other. The general colour of A is usually pale rose-madder.

Cassini's division always appears very dark indeed, but under the most favourable conditions it is never a perfect black comparable with the shadow of the globe on the ring.

B. From the outer zone of this ring, which is always very brilliant, its tone is usually softly and gently graduated to its very dark inner margin, without the slightest indication of marked variations. But on some occasions I have seen very slight deviations from this regular progression of tone. On Feb. 11, immediately within the brilliant outer zone was one slightly darker. But within that, the brightness very slightly increased again for a short space. And within that, the intensity of tone gradually increased to the inner margin. Under a high power the inner margin appears so very ill defined and blended with C, that it is not easy to determine the boundary between the two. The closest scrutiny has never enabled me to detect a division between or upon either. The general tint of this ring appears bluish-gray.

C. The general colour of this ring appears to me slaty-gray, but tinted with purple on the ansæ. Its fairly definite interior edge is probably rendered the more perceptible, by the sudden contrast of the black sky within the ansæ. Its general tone becomes gradually but very decidedly darker from its exterior to its interior margin, except where it crosses the globe. There the reverse is the case, for it becomes apparently lighter towards its interior margin, where its outline is very soft and diffused, as if semi-transparent. But its whole breadth is perceptible on the globe, and the globe is seen through the whole breadth of the ring, with the slight exception of a very narrow zone including its outer margin. It is remarkable that on Feb. 11, and other occasions, it appeared brighter as well as slightly narrower on the preceding ansa than on the following one, as I have represented in the drawing. When definition is very fine, a sort of grain is decidedly visible over its whole extent, as if its ultimate particles were aggregated in groups just large enough to be seen by momentary glimpses. But although this effect cannot be seen persistently, it certainly cannot be represented by a uniformly flat wash of colour.

Eccentricity of Rings in reference to Globe.

When the differing breadths of the two ends of C were observed on Feb. 11, the inner margin of the preceding ansa was seen to be slightly nearer to the globe than was that of the following one. And on Feb. 29, at 9^h, this eccentricity appeared on the same side, and to a much greater extent than I had ever observed. Again, on March 2, at 9^h, the eccentricity appeared about midway between its quantities on the two former dates.

Shadow of the Globe on the Rings.

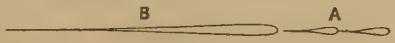
Several times during February I examined this carefully on account of its peculiar form. It appeared either to just meet or to cover Cassini's division, and followed it round towards the planet's polar limb without quite reaching it. The eastern margin of the shadow almost reproduced the outline of the globe, until near Cassini's division, when it curved sharply round towards the east and joined the division. No trace of the shadow was to be detected on the ring A.

On the probable Section of the Rings.

The extreme thinness of the rings, and the deviation of their surfaces from perfect planes, are matters of general observation.

The general form of those surfaces, as indicated by my observations of the shadow of the globe falling upon them in all positions of the ring, I have endeavoured to approximately determine, by critically examining in sunlight various models of the globe and rings which I had made to scale; the result being that *only one* form would yield (under the necessary conditions) artificial shadows in accord with all the real ones I have observed.

The section so obtained is here given.



On the Constitution of the Rings.

Observation and mathematical investigation alike indicate that the matter of which the rings are composed is in the condition of circulating streams of solid particles.

It has been matter of observation from the days of Short and Kater to the present time, that the outer ring A exhibits a more marked tendency to *striation* than the others. This I found very unusually visible on Nov. 22, 1881, and on different occasions since then. Similarly (only in recent times) the inner ring C has been found to exhibit the greatest tendency to *granulation*.

Observation has also shown (by comparison of the shadows of the globe) that the greatest thickness of each ring lies towards its outer margin.

To these results of observation may be added the consideration that the outer portion of the ring is doubtless occupied by its larger and heavier particles, and conversely, that its smaller and lighter ones compose its inner portion.

It then becomes interesting to note the following facts of observation:—

The maximum brightness coincides with the maximum thickness near the outer margin of B.

The maximum darkness of tone with the thinnest portion, which is the inner margin of C.

The greatest tendency to striation, with the place of the heaviest particles, as in A.

The greatest tendency to granulation, with the position probably occupied by the lightest particles, the inner ring C.

From this we may infer that the matter of which the ring B is composed, is less likely than that of the other rings, to develop distinctively either the character of granulation or of striation; and conclude that the gravitational influences which affect the ring system, produce slowly and continually all those variations of tone, tint, and structure which we are able to observe.

18 *Preston Street, Brighton.*

Faint Stars near Alcyone and near β^1 and β^2 Capricorni. By A. A. Common.

On the night of November 19, 1881, I examined the immediate neighbourhood of *Alcyone* with the three-foot Reflector and noted some faint stars. As they are in the centre of such an interesting group I have given a sketch showing their approximately relative positions to the well-known stars which are represented on the sketch by round dots, the position of the fainter stars being marked by crosses. The faintest of these is that nearly midway between two of the well-known three stars near *Alcyone*.

In August 1879, one of the first objects examined with this telescope, then just mounted, was the faint pair between β^1 and β^2 *Capricorni*, mentioned by Smyth in the *Bedford Catalogue*, p. 475, as having been discovered by Sir John Herschel, who considered it a delicate object, and remarked that no telescope

that was not competent to divide this pair would have the remotest chance with the satellite of *Uranus*.

While examining this pair I found another of a similar character, but very much fainter, within a radius of, say, 2′ of arc of the known pair, the two pairs forming a very delicate test for a large aperture. All these faint stars are easily seen—that is to say, they are not mere glimpse objects in this telescope—and it would therefore be interesting to know if some of the larger Refractors show them with the same ease. I have not given the exact place of the pair, so as not to mislead an observer, but I shall be glad to compare places.

1884 *June* 10.

On a New Variable Star in Monoceros. By J. E. Gore, M.R.I.A.

On the evening of February 17, 1884, while estimating the magnitude of 19 *Monocerotis*, suspected of variation by Dr. Gould (*Uranometria Argentina*, p. 332), I was surprised to find that one of the comparison stars selected—Lalande 13627—estimated $5{\cdot}6^m$ at Cordoba, was only 6^m, and more than half a magnitude fainter than 20 *Monocerotis*, rated 5·4 in the *Uranometria Argentina*. As Dr. Gould's magnitudes are so very accurate I immediately suspected variation, but, owing to cloudy weather, I could not confirm my observation until February 21, when I estimated the star in question only 2 steps brighter than Lalande 13706 (6·2 Gould) *nf*.

I then determined to watch the star for variation, and I give the results of my observations on every clear night up to April 20, 1884, which was the last occasion on which I could observe the star. From these observations it will be seen that the star increased in brightness up to $5{\cdot}5^m$ on March 4. It then slowly diminished in light, and was again only 6^m on March 21. After that date it again increased, and was about $5{\cdot}6^m$ on April 8 and 9, since which date it again diminished, and was about $5{\cdot}9^m$ when last observed on April 20. A small variation would therefore seem to be certain, with a period of about ± 35 days, but a more extended series of observations will of course be necessary to determine the exact period with accuracy.

The observed fluctuations of light would at first sight appear to be trifling, but we have a similar case in the well-known variable R (13) *Lyræ*, which only varies from 4·3 to 4·6, with a period of forty-six days, and in R *Eridani*, which varies from 5·4 to 6·0, according to the independent observations of three observers at Cordoba. Other cases of small variation are T *Carinæ* (6·2 to 6·9) and R *Muscæ* (6·6 to 7·4), discovered at Cordoba.

The star is reddish, and some of the observations may possibly have been slightly influenced by the presence of moonlight, although I do not find such to be the case in my observations of

the well-known variable μ *Cephei*, which is a remarkably red star.

The comparison stars are—

		Mag. Gould.
(A) =	20 Monocerotis	5·4
(B) =	Lalande 13627	5·6 supposed variable.
(C) =	„ 13339	5·9
(F) =	„ 13143	6·1
(D) =	„ 13706	6·2
(E) =	„ 13375	6·4
(G) =	„ 13104	6·2

19 *Monocerotis* I have variously estimated from 5·3 to 5·6 during the course of the following observations. I also suspect the star C of a very small fluctuation of light. The other comparison stars seem to be constant.

Closely *f* 19 *Monocerotis* is a small star, Lalande 13678 ($8\frac{1}{2}^m$), which was suspected of variation by Taylor. It was estimated 8^m at Cordoba, and Dr. Gould says, 'The supposition of variability may have arisen from mistaking it for F 19' (*Uranometria Argentina*, p. 333).

The place of Lalande 13627 is for 1884·0

	h m s
R.A.	6 56 15
	° ′ ″
Dec. S.	5 33 23

The observations were made by eye estimations with a binocular field-glass having object-glasses of 2 inches aperture, and a magnifying power of about 6 diameters.

Observations of Lalande 13627.

Date of Obs.	d	h	Est. Mag.	Remarks.
1884, Feb.	17		6·0	Very little brighter than D; more than $\frac{1}{2}$ mag. less than 20.
	21		6·0	2 steps brighter than D, but 6 steps less than 20 Monocerotis.
Mar.	3		5·8	Moonlight; midway in light between 20 and D.
	4	10	5·5	Only 1 step less than 20 Monocerotis.
	7	8	5·6	Moonlight.
	9	$9\frac{1}{2}$	5·7	„
	10	$9\frac{1}{2}$	5·7	Strong moonlight; observation difficult.
	11	8	5·75	Strong moonlight and hazy sky.
	13	$10\frac{3}{4}$	5·8	„ „

Date of Obs.			Est. Mag.	Remarks.
	d	h		
1884, Mar.	14	8	5·85	No moon.
	15	8¼	5·9	„
	17	8¾	5·9	Fine night; no moon.
	18		5·8	Hazy sky.
	20	10¾	5·9	No moon.
	21		6·0	„
	23	8¼	5·9	„
	24	9½	5·8	Hazy sky.
	25			On this date the star was estimated 5·8 by Franks at Leicester with 11¼-in. reflector.
	31	8¼	5·7	Hazy sky and moonlight.
Apr.	1		5·7	
	2		5·7	Strong and near moonlight.
	6	10½	5·7	Strong moonlight and star low.
	7		5·65	„ „
	8	8½	5·6	Strong moonlight.
	9	8¼	5·6	„
	10	9	5·7	„
	11	8¾	5·7	Hazy sky.
	12	9	5·7	Clear sky; no moon.
	17	9	5·8	No moon.
	20	9½	5·9	No moon; star near W. horizon.

Suggestions for the Improvement of the Transit Circle.

By Ralph Copeland, Ph.D.

In a recent paper containing suggestions for the construction of a large Transit Circle Mr. Common has mainly dealt with the stiffness and intimate connection of an axis and telescope of steel, contenting himself as regards the circle either with making it out of a single steel disk separately attached to the axis or forming part of the frame or girder by which it is proposed to replace the tube. As a whole Mr. Common's scheme may therefore be described as embracing a telescope and axis of enormous strength, obtained by the use of considerable quantities of steel, while all desirable mobility and relief from strain, in the axis at least, are to be secured by the use of mercurial counterpoises. Mr. Common's plans do not profess to go into detail, and thus leave in a great measure untouched several points in the construction of our larger Transit Circles that are perhaps quite as great sources of error as any want of

stiffness in the axis. I think few people can contemplate a large Transit Circle with its objective cell held by sets of opposing centring screws, its eye end with movable Right Ascension wire plate crossed by a declination micrometer, and these in their turn, followed by "slipping" slides carrying the eyepiece across the field, perhaps in Declination as well as Right Ascension —few people, I think, can look at this accumulation of mechanism without asking if it really permits the wires to maintain their zero points with regard to the optical axis unaltered in all positions of the instrument. In the case of parallax or other delicate observations with the equatorial this would not be assumed for one moment; all the changes would be rung of telescope preceding, telescope following, micrometer screw north, and micrometer screw south. These are the precautions that would be taken in merely differential operations. They should hardly be disregarded in fundamental observations. Many of the risks of instrumental derangement alluded to can be avoided in a great measure by letting the object-glass directly into one end of the tube and the wire plate into the other. By continually testing the centring and focus during the progress of the work this need not be a difficult operation.* Something would doubtless also be gained by selecting a material for the tube such that its coefficient of expansion would be the same as that of the *focal length* of the object glass, a precaution taken many years ago by the late Prof. Bruhns in the construction of the Leipzig Refractor. Fraunhofer's well-known plan of securing the objective as described in *Ast. Nach.*, No. 59, is still in general use; it would be rendered more certain in its action by cementing the lenses together. The use of a perfectly plain, unbroken and unpierced cylindrical tube passing through and *soldered into* an axis sufficiently enlarged in the middle of its length was suggested to me by my friend Dr. David Gill in conversation some years ago. This promising device has the great advantage of extreme simplicity. The telescope thus reduced to its simplest elements of an object-glass, the fiducial wires (no matter whether these are spiders' webs or lines ruled on glass), and a plain supporting tube ought to be considered as something not further to be loaded or otherwise molested. I therefore propose to surround the same by an outer separate tube, deriving its support from the neighbourhood of the *point d'appui* of the counterpoises, which should carry all the illuminating apparatus, the eyepiece and its slides, as well as any micrometer that might be found desirable. Advantage might also be taken of such an arrangement to protect the tube from all sudden changes of temperature caused by currents of air or by radiation from the observer's body, particularly when making reflex and nadir observations. The oft-repeated proposal of using a glass disk in place of a

* Mr. Common's proposal to use a double telescope "where the O.G. of one is in front of the eyepiece of the other" would here be of the greatest practical value. See p. 293.

graduated metal wheel offers several advantages beyond those usually claimed, chief among which may be reckoned the facility of illumination *through* the circle and the facility of adjusting the tubes of the microscopes perpendicular to its plane by means of reflections from its surface. The divisions could doubtless also be made much finer than on silver, which, for instance, in the Dunecht instrument are about 7″ in width.

Another point connected with the divided circles does not seem to have received all the attention it deserves. It is well known that many of the finer circles in use are so accurate that their division errors do not exceed the deformations to which they may be readily exposed by inequality of temperature in their different parts; for in a brass circle an arc of 60° is expanded about 1″ for 1° Fahr. Hence a difference of temperature in the upper and lower parts of a perfect circle to this extent would produce apparent division errors of very appreciable magnitude. It is to be feared that in many observatories much larger differences of temperature may be found in the neighbourhood of the Transit Circle at points not more distant from each other than the circle's diameter. In vol. i. of the *Annalen der Sternwarte in Leiden*, p. ciii., may be found an interesting account of the precautions observed by Pistor and Martins when engaged in graduating a circle, from which we learn that when a circle is mounted on the dividing engine the surrounding air is kept in motion by automatic fans for several hours before a single division is cut, and the circulation of the air is maintained as long as the work is in progress. If this is found necessary in the construction of a circle it is no less necessary in its most refined use. Perhaps by enclosing the circle in a box in which the air is kept in motion by a light fan driven by clockwork the chief requirements of the case would be met. Perhaps it might be found advantageous to surround the box by some bad conductor of heat. It is at least certain that no experimentalist would compare two thermometers immersed in a fluid without carefully stirring the same. In examining the division errors of one of the circles of the Dunecht meridian instrument it was found that the interposition of a milled board between the circle and the massive cast-iron pier head greatly improved the accordance of the observations, probably by cutting off the radiations from the pier. In the case, too, of reflex observations it has long been known that imperfect ventilation of the neighbourhood of the quicksilver trough seriously affected the value of the results. Here again some form of mechanical aid to ventilation seems to be called for. It is very desirable also that all the illuminating lamps should be banished once for all from the meridian room and its more immediate vicinity. By passing the needful beams of light through water they could be deprived of nearly all their heating effect.

Dunecht Observatory:
1884 *June* 10.

Note on the Causes of the Blurred Patches in Instantaneous Photographs of the Sun. By A. C. Ranyard.

The beautiful photographs of the sun taken at Meudon by M. Janssen exhibit curious patches of a lighter shade within which the definition of the rice grain structure of the photosphere is interfered with. M. Janssen is inclined to attribute these patches to solar clouds or gaseous masses above the photosphere, which he supposes change their shape very rapidly. I have recently been making some experiments which show that similar indistinct lighter patches may be caused by currents of heated air in the tube of the telescope. The experiments have been made with a reflecting telescope with a heavy iron tube. If the image of a bright star is observed out of focus on an ordinary night bright ripples may be seen passing across the disk of light, which is really an image of the speculum with the flat projected on its centre. That these ripples are due to the unequal refraction of heated air currents may be proved by placing a heated body in the tube of the telescope. I made use of a heated iron chisel, which was placed in different parts of the mouth of the telescope, and was let down the tube by a string. This greatly increased the distinctness of the ripples, as well as the velocity with which they passed across the image of the speculum; the dark lines corresponding to the supports of the flat enabled one to localize the part of the tube from which the heated air was rising, and showed that it corresponded with the place where the hot piece of iron was held. Though these ripples are only seen when the star is out of focus, the differences of refraction caused by heated air currents must affect the sharpness of the image in focus, and give rise to scattered light round the image of a star. In the case of a uniformly bright disk heated air currents would evidently give rise to areas of greater and less brightness. In order to test this I took some instantaneous photographs of the sun with a piece of heated iron in the mouth of the telescope, and also when the sun was seen just above a heated roof. In both cases the limb of the sun as shown in the photographs is much distorted, and brighter areas are seen extending over the sun's disk. In the case of the photographs taken with the hot iron in the mouth of the telescope the brighter areas have a tree-like form.

The photographs were taken in a camera 7 ft. 6 in. long, attached to the side of an 18-in. telescope of 9 ft. focal length. The light is reflected from the flat of the telescope as in the ordinary Newtonian form, and again reflected by a second flat into the camera, which is provided with a photographic lens giving an image of the sun nearly thirty inches in diameter.

Observations of Occultations of Stars by the Moon, 1883–4, and of Phenomena of Jupiter's Satellites, 1884, made at the Radcliffe Observatory, Oxford.

(*Communicated by E. J. Stone, M.A., F.R.S., Radcliffe Observer.*)

Occultations of Stars by the Moon.

Ref.	Day of Observation.	Phenomenon.	Moon's Limb.	Instrument.	Power.	G.M.T. of Obs. h m s	Observer.
(*a*)	1883, July 17	Disapp. B.A.C. 6081	Dark	Heliometer	200	11 28 52·07	W.
(*b*)	Aug. 24	Reapp. B.A.C. 1206	,,	42-inch Telescope	80	11 21 59·97	R.
(*c*)	Nov. 19	,, A^2 Cancri	,,	,,	80	12 18 38·44	R.
(*d*)	1884, Mar. 6	Disapp. λ Geminorum	,,	10-foot Equatorial	160	10 7 35·24	R.
(*e*)	6	Reapp. λ Geminorum	Bright	,,	160	11 14 6·97	W.

Remarks.

(*a*) Sky hazy and star very faint.
(*b*) and (*c*) Observations good.
(*d*) Instantaneous; observation good.
(*e*) Appeared gradually, taking about one second to acquire full brightness.

Phenomena of Jupiter's Satellites.

Ref.	Day of Observation.	Satellite.	Phenomenon.	Instrument.	Power.	G.M.T. of Obs. h m s	G.M.T. of N.A. h m s	Observer.
	1884, Feb. 13	III.	Tr. E. First contact	10-foot Equatorial	160	7 28 34	7 35 0	R.
			Bisection	,,	,,	7 32 59		
			Last contact	,,	,,	7 37 48		
(*a*)	15	I.	Ecl. R. First seen	,,	,,	12 5 20	12 5 28	,,
(*b*)	21	II.	Ecl. R. First seen	,,	,,	8 16 45	8 16 9	F.B.

Ref.	Day of Observation.	Satellite.	Phenomenon.	Instrument.	Power.	G.M.T. of Obs. h m s	G.M.T. of N.A. h m s	Observer.
	1884, Mar. 17	I.	Tr. I. Ext. contact	10-foot Equatorial	160	8 5 27		
			Bisection	"	"	8 9 26	8 8 0	R.
			Last contact	"	"	8 13 56		
(*b'*)	18	I.	Ecl. R. First seen	"	"	8 43 20	8 43 27	F.B.
	21	IV.	Ecl. R. Suspected	"	125	8 17 32	8 22 19	R.
			Decided	"	"	8 17 47		
			Full brightness	"	"	8 33 44		"
(*c*)	21	IV.	Ecl. R. First trace	Heliometer	160	8 17 36	8 22 19	F.B.
			Fully established	"	"	8 17 44		
			Full brightness	"	"	8 35 9		"
	24	II.	Ecl. R. First seen	10-foot Equatorial	125	7 56 3	7 56 6	R.
(*d*)			As bright as J.I.	"	"	7 59 38		"
			Full brightness	"	"	8 1 38		"
	24	II.	Ecl. R. As bright as J.I.	Heliometer	160	7 59 51		F.B.
(*d'*)	24	I.	Tr. I. Ext. contact	10-foot Equatorial	125	9 57 19		
			Bisection	"	"	10 1 3	9 59 0	R.
			Int. contact	"	"	10 5 18		
(*e*)	Apr. 10	I.	Ecl. R. First seen	Heliometer	160	8 58 52	8 58 36	W.
(*f*)	10	I.	Ecl. R. First seen	10-foot Equatorial	125	8 59 20	8 58 36	F.B.
			Full brightness	"	"	9 0 50±		"
(*g*)	10	III.	Tr. I. First contact	Heliometer	160	9 6 41		
			Bisection	"	"	9 13 50	9 12 0	W.
			Last contact	"	"	9 21 39		

Ref.	Day of Observation.	Satellite.	Phenomenon.	Instrument.	Power.	G.M.T. of Obs. h m s	G.M.T. of N.A. h m s	Observer.
	1884, Apr. 10	III.	Tr. I. First contact	10-foot Equatorial	125	9 8 16		
			Bisection	„	„	9 13 25	9 12 0	F.B.
			Last contact	„	„	9 19 54		
(*h*)	21	III.	Ecl. D. Last seen	Heliometer	160	8 25 13	8 23 10	„
(*i*)	21	III.	Ecl. R. First seen	10-foot Equatorial	125	11 54 37	11 53 55	R.
			Half as bright as J.I.	„	„	11 57 32		„
			As bright as J.I.	„	„	11 59 32		„
			Full brightness	„	„	12 0 18		„
(*k*)	May 5	III.	Occ. D. First contact	„	125	11 13 49		
			Bisection	„	„	11 18 48	11 23 0	„
			Last contact	„	„	11 23 17		
(*l*)	10	I.	Ecl. R. First seen	„	„	11 9 21	11 9 21	F.B.

Remarks.

(*a*), (*b*), (*c*), (*d'*), (*f*), and (*l*). Images good. (*b'*) The time noted has been increased by 5^m.
(*d*) At this time the two satellites (I. and II.) appeared like a very close double star of equal magnitudes.
(*e*) May be a second or two late. (*g*) Image very good at times. (*h*) Good; disappeared quicker than usual.
(*i*) Time of first seen very probably late. Some light lost; *Jupiter* and his satellites observed through trees. Not satisfactory.
(*k*) *Jupiter's* limb very ill-defined. Satellite rather faint.

In converting Oxford Mean Solar Times into Greenwich Mean Solar Times the following Longitude has been used: 5^m 2^s·6 W.

Observers: W. = Mr. Wickham. R. = Mr. Robinson. F.B. = Mr. Bellamy.

Ephemerides of the Satellites of Saturn, 1884–85.

By A. Marth.

The following ephemerides of the five inner satellites are founded upon the elements which have served in the preparation of the ephemerides for the last apparitions of *Saturn*, as the results of some observed conjunctions of the satellites with the centre of the planet indicate that the adopted elements are sufficiently near the truth for the present purpose. The five satellites deviate so little from the plane of the ring that it will be most suitable to treat their deviations as latitudes above this plane, the ascending node N and inclination I of which in reference to the plane of the Earth's equator are here assumed to be for 1885·0

$$N = 126^\circ 5128 \qquad I = 7^\circ 0059.$$

The assumed longitudes of the satellites in their orbits (*i.e.* their longitudes from the ascending node added to the Right Ascension N of the ascending node reckoned from the true point of the equinox) referred to the time when the light arrives at the distance [0·950], or at the distance the logarithm of which is 0·950, are the following:

0^h Greenwich.	Mimas.	Enceladus.	Tethys.	Dione.	Rhea.
1884, Aug. 21	126°501	89°357	93°634	275°252	157°529
Sept. 20	66·244	51·314	54·578	261·304	28·234
Oct. 20	5·989	13·270	15·522	247·355	258·938
Nov. 19	305·735	335·227	336·467	233·407	119·643
Dec. 19	245·482	297·183	297·411	219·459	0·347
1885, Jan. 18	185·231	259·140	258·356	205·511	231·052
Feb. 17	124·980	221·097	219·300	191·562	101·756
Mar. 19	64·700	183·053	180·245	177·614	332·461
Apr. 18	4·481	145·010	141·189	163·665	203·165

The adopted daily sidereal motions of the five satellites and the corresponding times of their sidereal revolutions are:

		d h m s
Mimas	381°99146	0 22 37 5·91
Enceladus	262·73186	1 8 53 6·86
Tethys	190·69812	1 21 18 25·96
Dione	131·53503	2 17 41 9·33
Rhea	79·69012	4 12 25 11·87

The values for *Mimas* belong to Sept. 20, 1884, and depend on the hypothesis of accelerated motion introduced in the Ephemeris for 1880.

In the following tables P denotes the position-angle of the

minor axis of the ring; L+180° the planetocentric longitude of the Earth referred to the plane of the ring; Λ+180° that of the Sun, or Λ−L the difference between the two. The apparent equatorial diameter of the ball and the diameter of the outer rim of the ring depend still on Bessel's determinations. The assumed proportion of the polar axis of the ball to the equatorial diameter is=0·900.

In the tables for the satellites *a* and *b* are the semi-axes of the apparent orbits, *l*−L the longitudes of the satellites in their orbits reckoned from the points which are in superior conjunction with the planet's centre or in opposition to the Earth in longitude. By adding to *l*−L the value of L from the first table the longitudes *l* are found.

The values of P, *a*, *b* and *l*−L are to be interpolated for the times for which the apparent positions of the satellites are required, and the rectangular coordinates *x* and *y*, reckoned parallel to the axes of the ring and expressed in seconds of arc, or, if polar coordinates are wanted, the position-angles *p* and distances *s* of the satellites in reference to the centre of the planet are then found by means of the equations

$$s \sin (p-\mathrm{P}) = x = a \sin (l-\mathrm{L})$$
$$s \cos (p-\mathrm{P}) = y = b \cos (l-\mathrm{L}).$$

h Gr.	P.	L.	Latitude of Earth above plane of ring.	Latitude of Sun above plane of ring.	Λ−L.
1884.	°	°	°	°	°
Aug. 31	354·361	84·892	−26·615	−26·697	−6·933
Sept. 5	·332	85·215	·595	·705	7·048
10	·308	85·492	·576	·713	7·117
15	·288	85·723	·559	·721	7·139
20	·273	85·906	·543	·728	7·113
25	·262	86·039	·530	·735	7·038
30	·254	86·122	·519	·742	6·912
Oct. 5	354·252	86·153	−26·511	−26·749	−6·734
10	·254	86·133	·506	·755	6·505
15	·261	86·061	·504	·761	6·225
20	·272	85·939	·505	·767	5·894
25	·287	85·766	·509	·772	5·513
30	·307	85·546	·516	·777	5·084
Nov. 4	354·330	85·282	−26·525	−26·782	−4·611
9	·357	84·976	·536	·786	4·096
14	·388	84·631	·550	·790	3·542
19	·423	84·253	·565	·794	2·955
24	·460	83·847	·582	·798	2·339
29	·499	83·418	·599	·801	1·701

0^h Gr.	P.	L.	Latitude of Earth above plane of ring.	Latitude of Sun above plane of ring.	A−L.
1884.	°	°	°	°	°
Dec. 4	354·540	82·971	−26·617	−26·804	−1·046
9	·583	82·515	·635	·806	−0·380
14	·626	82·054	·653	·808	+0·290
19	·670	81·596	·671	·809	0·957
24	·712	81·147	·689	·810	1·615
29	·754	80·714	·707	·811	2·257
1885.					
Jan. 3	354·794	80·305	−26·724	−26·812	+2·876
8	·831	79·923	·741	·813	3·467
13	·865	79·574	·758	·813	4·025
18	·895	79·262	·775	·814	4·546
23	·921	78·994	·792	·813	5·024
28	·943	78·771	·809	·812	5·456
Feb. 2	354·960	78·596	−26·827	−26·811	+5·840
7	·972	78·471	·845	·810	6·174
12	·979	78·398	·863	·809	6·456
17	·980	78·378	·882	·808	6·685
22	·976	78·412	·901	·806	6·861
27	·967	78·498	·920	·804	6·984
Mar. 4	354·953	78·636	−26·939	−26·801	+7·055
9	·933	78·825	·958	·798	7·075
14	·908	79·063	·976	·795	7·047
19	·879	79·348	26·994	·791	6·971
24	·846	79·680	27·011	·787	6·847
29	·808	80·058	27·026	·783	6·679
Apr. 3	354·767	80·477	−27·039	−26·779	+6·471
8	·722	80·931	27·049	26·775	6·225

0^h Gr.	Diameter of Ball. Equat.	Phase prec. *l*.	Polar	Axis of Ring. Major	Minor	*Mimas.* a_1	b_1	l_1−L.	Diff.
1884.	″	″	″	″	″	″	″	°	°
Aug. 31	17·60	0·066	16·21	40·57	18·17	27·68	−12·40	260·79	1909·81
Sept. 5	17·76	·068	16·35	40·93	18·32	27·92	12·50	10·60	·86
10	17·92	·069	16·50	41·30	18·48	28·17	12·60	120·46	·91
15	18·08	·070	16·65	41·68	18·64	28·43	12·71	230·37	1909·96
20	18·25	·070	16·80	42·06	18·80	28·69	12·82	340·33	1910·01
25	18·42	·069	16·96	42·46	18·96	28·96	12·94	90·34	·05
30	18·59	·068	17·12	42·85	19·13	29·23	13·05	200·39	·10

0^h Gr.	Diameter of Ball. Equat.	Phase prec. *l.*	Polar	Axis of Ring. Major	Minor	*Mimas.* a_1	b_1	$l_1-L.$	Diff.
1884.									
Oct. 5	18″·76	″·065	17″·27	43″·24	19″·30	29″·50	−13″·17	310°·49	1910·14
10	18·93	·061	17·43	43·63	19·47	29·76	13·28	60·63	·19
15	19·09	·056	17·58	44·01	19·64	30·02	13·40	170·82	·24
20	19·25	·051	17·73	44·38	19·81	30·27	13·51	281·06	·28
25	19·41	·045	17·87	44·73	19·97	30·51	13·62	31·34	·32
30	19·55	·039	18·00	45·07	20·12	30·74	13·72	141·66	·35
Nov. 4	19·68	·032	18·13	45·38	20·27	30·95	−13·82	252·01	1910·37
9	19·81	·025	18·24	45·66	20·40	31·15	13·92	2·38	·40
14	19·92	·019	18·34	45·91	20·52	31·32	14·00	112·78	·42
19	20·01	·013	18·43	46·13	20·63	31·47	14·07	223·20	·43
24	20·09	·008	18·50	46·31	20·72	31·59	14·13	333·63	·44
29	20·15	·004	18·55	46·44	20·79	31·68	14·18	84·07	·44
Dec. 4	20·19	·002	18·59	46·53	20·85	31·74	−14·22	194·51	1910·43
9	20·21	·000	18·61	46·58	20·88	31·78	14·25	304·94	·42
14	20·21	foll. l.	18·61	46·58	20·90	31·78	14·26	55·36	·40
19	20·19	·001	18·59	46·54	20·89	31·75	14·25	165·76	·38
24	20·15	·004	18·56	46·45	20·86	31·69	14·23	276·14	·34
29	20·09	·008	18·51	46·32	20·82	31·60	14·20	26·48	·30
1885.									
Jan. 3	20·02	·013	18·44	46·14	20·75	31·48	−14·15	136·78	1910·25
8	19·92	·018	18·35	45·93	20·66	31·33	14·10	247·03	·21
13	19·81	·024	18·25	45·68	20·56	31·16	14·03	357·24	·16
18	19·69	·031	18·14	45·40	20·45	30·97	13·95	107·40	·10
23	19·56	·038	18·02	45·09	20·32	30·75	13·86	217·50	1910·04
28	19·41	·044	17·88	44·75	20·18	30·52	13·77	327·54	1909·98
Feb. 2	19·26	·050	17·74	44·39	20·03	30·28	−13·67	77·52	1909·93
7	19·10	·055	17·59	44·02	19·88	30·03	13·56	187·45	·87
12	18·93	·060	17·44	43·64	19·72	29·77	13·45	297·32	·81
17	18·76	·064	17·29	43·25	19·56	29·50	13·34	47·13	·75
22	18·59	·067	17·13	42·85	19·39	29·23	13·23	156·88	·69
27	18·42	·068	16·97	42·46	19·22	28·96	13·11	266·57	·64

0h Gr.	Diameter of Ball. Equat.	Phase prec. l.	Polar.	Axis of Ring. Major	Minor	*Mimas.* a_1	b_1	l_1-L.	Diff.
1884.	″	″	″	″	″	″	″	°	°
Mar. 4	18·25	·069	16·81	42·06	19·06	28·69	−13·00	16·21	1909·58
9	18·08	·069	16·66	41·68	18·89	28·43	12·89	125·79	·54
14	17·91	·068	16·51	41·30	18·73	28·17	12·78	235·33	·49
19	17·75	·066	16·36	40·93	18·58	27·92	12·67	344·82	·45
24	17·60	·063	16·22	40·57	18·42	27·67	12·57	94·27	·41
29	17·45	·059	16·08	40·22	18·28	27·44	12·47	203·68	·37
Apr. 3	17·31	·055	15·95	39·89	18·14	27·21	−12·37	313·05	1909·34
8	17·17	0·051	15·82	39·58	18·00	27·00	12·28	62·39	

	Enceladus. a_2	b_2	l_2-L	Diff.	*Tethys.* a_3	b_3	l_3-L	Diff.
1884.	″	″	°	°	″	″	°	°
Aug. 31	35·51	−15·90	111·28	1313·46	43·95	−19·69	115·36	953·26
Sept. 5	35·82	16·03	344·74	·50	44·34	19·85	348·62	·30
10	36·14	16·17	218·24	·56	44·74	20·02	221·92	·35
15	36·47	16·31	91·80	·60	45·15	20·19	95·27	·40
20	36·81	16·45	325·40	·65	45·57	20·36	328·67	·45
25	37·15	16·59	199·05	·70	45·99	20·54	202·12	·49
30	37·50	16·74	72·75	·75	46·42	20·73	75·61	·55
Oct. 5	37·84	−16·89	306·50	1313·80	46·84	−20·91	309·16	953·59
10	38·18	17·04	180·30	·84	47·26	21·09	182·75	·65
15	38·51	17·19	54·14	·89	47·68	21·28	56·40	·69
20	38·84	17·33	288·03	·93	48·08	21·46	290·09	·74
25	39·15	17·47	161·96	1313·97	48·46	21·63	163·83	·78
30	39·44	17·61	35·93	1314·01	48·82	21·80	37·61	·82
Nov. 4	39·71	−17·73	269·94	1314·05	49·16	−21·95	271·43	953·85
9	39·96	17·85	143·99	·07	49·47	22·10	145·28	·88
14	40·18	17·96	18·06	·09	49·74	22·23	19·16	·91
19	40·37	18·05	252·15	·11	49·97	22·35	253·07	·93
24	40·52	18·13	126·26	·13	50·16	22·45	127·00	·95
29	40·64	18·20	0·39	·13	50·31	22·53	0·95	·95
Dec. 4	40·72	−18·24	234·52	1314·13	50·41	−22·59	234·90	953·96
9	40·76	18·27	108·65	·12	50·46	22·62	108·86	·95
14	40·77	18·29	342·77	·11	50·46	22·64	342·81	·94
19	40·73	18·28	216·88	·08	50·42	22·63	216·75	·92
24	40·65	18·26	90·96	·06	50·32	22·60	90·67	·90
29	40·53	18·22	325·02	1314·02	50·17	22·55	324·57	·87

		Enceladus.				*Tethys.*			
		a_2	b_2	l_2-L	Diff.	a_3	b_3	l_3-L	Diff.
1884.		″	″	°	°	″	″	°	°
Jan.	3	40·38	−18·16	199·04	1313·99	49·98	−22·48	198·44	953·83
	8	40·19	18·08	73·03	·94	49·75	22·39	72·27	·79
	13	39·97	18·00	306·97	·89	49·48	22·28	306·06	·75
	18	39·72	17·90	180·86	·84	49·17	22·15	179·81	·70
	23	39·45	17·78	54·70	·79	48·84	22·01	53·51	·64
	28	39·16	17·66	288·49	·73	48·48	21·86	287·15	·59
Feb.	2	38·85	−17·53	162·22	1313·68	48·09	−21·70	160·74	953·54
	7	38·52	17·40	35·90	·62	47·69	21·53	34·28	·48
	12	38·19	17·26	269·52	·56	47·27	21·36	267·76	·43
	17	37·85	17·11	143·08	·50	46·85	21·18	141·19	·37
	22	37·50	16·97	16·58	·45	46·42	21·00	14·56	·31
	27	37·15	16·82	250·03	·40	45·99	20·82	247·87	·26
Mar.	4	36·81	−16·68	123·43	1313·35	45·57	−20·64	121·13	953·21
	9	36·47	16·53	356·78	·29	45·15	20·47	354·34	·16
	14	36·14	16·39	230·07	·25	44·73	20·29	227·50	·12
	19	35·81	16·26	103·32	·21	44·33	20·12	100·62	·07
	24	35·50	16·12	336·53	·16	43·95	19·96	333·69	953·02
	29	35·20	15·99	209·69	·13	43·57	19·80	206·71	952·99
Apr.	3	34·91	−15·87	82·82	1313·09	43·22	−19·65	79·70	952·96
	8	34·64	15·75	315·91		42·88	19·50	312·66	

		Dione.				*Rhea.*			
		a_4	b_4	l_4-L	Diff.	a_5	b_5	l_5-L	Diff.
1884.		″	″	°	°	″	″	°	°
Aug.	31	56·29	−25·21	65·46	657·41	78·61	−35·21	149·39	398·16
Sept.	5	56·79	25·42	2·87	·46	79·30	35·50	187·55	·21
	10	57·30	25·63	300·33	·51	80·02	35·80	225·76	·26
	15	57·82	25·85	237·84	·55	80·75	36·11	264·02	·31
	20	58·36	26·08	175·39	·61	81·50	36·42	302·33	·36
	25	58·90	26·31	113·00	·65	82·26	36·74	340·69	·40
	30	59·45	26·54	50·65	·71	83·02	37·07	19·09	·45
Oct.	5	59·99	−26·78	348·36	657·75	83·78	−37·40	57·54	398·51
	10	60·53	27·02	286·11	·81	84·53	37·73	96·05	·56
	15	61·06	27·25	223·92	·85	85·27	38·06	134·61	·60
	20	61·57	27·48	161·77	·90	85·99	38·38	173·21	·65
	25	62·06	27·70	99·67	·94	86·67	38·69	211·86	·70
	30	62·53	27·92	37·61	657·98	87·32	38·98	250·56	·75

		Dione.				*Rhea.*			
		a_4	b_4	l_4-L	Diff.	a_5	b_5	l_5-L	Diff.
1884.		"	"	°	°	"	"	°	°
Nov.	4	62·96	−28·12	335·59	658·02	87·92	−39·26	289·31	398·78
	9	63·35	28·30	273·61	·05	88·47	39·52	328·09	·81
	14	63·70	28·47	211·66	·09	88·96	39·76	6·90	·85
	19	64·00	28·62	149·75	·10	89·38	39·97	45·75	·87
	24	64·25	28·75	87·85	·12	89·72	40·15	84·62	·89
	29	64·43	28·85	25·97	·14	89·98	40·29	123·51	·91
Dec.	4	64·56	−28·93	324·11	658·14	90·16	−40·40	162·42	398·91
	9	64·63	28·97	262·25	·13	90·25	40·46	201·33	·91
	14	64·63	28·99	200·38	·13	90·26	40·49	240·24	·90
	19	64·57	28·98	138·51	·11	90·17	40·48	279·14	·90
	24	64·45	28·95	76·62	·09	90·00	40·42	318·04	·87
	29	64·26	28·88	14·71	·06	89·74	40·33	356·91	·85
1885.									
Jan.	3	64·02	−28·79	312·77	658·03	89·40	−40·20	35·76	398·81
	8	63·72	28·67	250·80	657·99	88·99	40·04	74·57	·78
	13	63·37	28·53	188·79	·95	88·50	39·85	113·35	·74
	18	62·98	28·37	126·74	·90	87·95	39·62	152·09	·69
	23	62·55	28·19	64·64	·85	87·35	39·37	190·78	·65
	28	62·09	28·00	2·49	·80	86·70	39·10	229·43	·59
Feb.	2	61·59	−27·80	300·29	657·75	86·01	−38·82	268·02	398·54
	7	61·08	27·58	238·04	·69	85·30	38·52	306·56	·49
	12	60·55	27·36	175·73	·64	84·55	38·21	345·05	·44
	17	60·00	27·13	113·37	·58	83·80	37·89	23·49	·38
	22	59·46	26·90	50·95	·52	83·03	37·57	61·87	·33
	27	58·91	26·67	348·47	·48	82·26	37·24	100·20	·27
Mar.	4	58·36	−26·44	285·95	657·42	81·50	−36·92	138·47	398·22
	9	57·82	26·21	223·37	·38	80·75	36·61	176·69	·18
	14	57·29	25·99	160·75	·32	80·01	36·30	214·87	·13
	19	56·78	25·77	98·07	·28	79·29	35·99	253·00	·08
	24	56·28	25·56	35·35	·24	78·60	35·70	291·08	·03
	29	55·81	25·36	332·59	·20	77·93	35·41	329·11	398·00
Apr.	3	55·35	−25·16	269·79	657·17	77·30	−35·14	7·11	397·96
	8	54·92	24·98	206·96		76·69	34·88	45·07	

Approximate Greenwich Mean Times of conjunctions of the satellites with the centre of *Saturn*:

"n" inferior conjunction with centre, or satellite exactly in the direction of the minor axis of the ring, north, moving from the following to the preceding side.
"s" superior conjunction, or satellite south, moving from the preceding to the following side.

1884.	h		1884.	h		1884.	h	
Aug. 31	8·1	Te. n.	Sept. 15	10·7	Te. n.	Sept. 24	21·2	Te. n.
	9·2	Rh. n.		22·3	Di. s.		22·3	En. n.
	20·9	Di. n.	16	4·9	Rh. s.	25	5·8	Rh. s.
Sept. 1	6·8	Te. s.		9·3	Te. s.		12·2	Di. n.
2	5·5	Te. n.		16·9	En. n.		14·7	En. s.
	5·8	Di. s.	17	7·1	Di. n.		16·9	Mi. s.
	15·5	Rh. s.		8·0	Te. n.		19·9	Te. s.
3	4·1	Te. s.	18	6·6	Te. s.	26	15·6	Mi. s.
	14·6	Di. n.		11·1	Rh. n.		18·5	Te. n.
4	2·8	Te. n.		15·3	Mi. n.		21·1	Di. s.
	21·7	Rh. n.		16·0	Di. s.	27	12·0	Rh. n.
	23·5	Di. s.		18·3	En. s.		14·2	Mi. s.
5	1·4	Te. s.	19	5·3	Te. n.		16·0	En. n.
6	0·1	Te. n.		10·7	En. n.		17·2	Te. s.
	8·3	Di. n.		13·9	Mi. n.	28	5·9	Di. n.
	22·7	Te. s.	20	0·8	Di. n.		8·5	En. s.
7	4·0	Rh. s.		3·9	Te. s.		12·8	Mi. s.
	17·2	Di. s.		12·5	Mi. n.		15·8	Te. n.
	21·4	Te. n.		17·4	Rh. s.	29	11·4	Mi. s.
8	20·1	Te. s.		19·6	En. n.		14·5	Te. s.
9	2·0	Di. n.	21	2·6	Te. n.		14.8	Di. s.
	10·2	Rh. n.		9·7	Di. s.		17·4	En. s.
	18·7	Te. n.		11·2	Mi. n.		18·2	Rh. s.
10	10·9	Di. s.		12·0	En. s.	30	9·8	En. n.
	17·4	Te. s.	22	1·2	Te. s.		10·0	Mi. s.
11	16·0	Te. n.		18·5	Di. n.		13·1	Te. n.
	16·4	Rh. s.		20·9	En. s.		23·6	Di. n.
	19·7	Di. n.		21·1	Mi. s.	Oct. 1	11·8	Te. s.
12	14·7	Te. s.		23·6	Rh. n.		18·7	En. n.
13	4·6	Di. s.		23·9	Te. n.		19·9	Mi. n.
	13·3	Te. n.	23	13·4	En. n.	2	0·5	Rh. n.
	22·7	Rh. n.		19·7	Mi. s.		8·4	Di. s.
14	12·0	Te. s.		22·6	Te. s.		10·4	Te. n.
	13·4	Di. n.	24	3·4	Di. s.		11·1	En. s.
	15·6	En. s.		18·3	Mi. s.		18·6	Mi. n.

1884.	h		
Oct. 3	9·1	Te.	s.
	17·2	Mi.	n.
	17·3	Di.	n.
	20·0	En.	s.
4	6·7	Rh.	s.
	7·7	Te.	n.
	12·4	En.	n.
	15·8	Mi.	n.
5	2·1	Di.	s.
	6·4	Te.	s.
	14·4	Mi.	n.
	21·3	En.	n.
6	5·0	Te.	n.
	11·0	Di.	n.
	12·9	Rh.	n.
	13·0	Mi.	n.
	13·8	En.	s.
7	3·7	Te.	s.
	19.8	Di.	s.
8	2·3	Te.	n.
	19·1	Rh.	s.
9	1·0	Te.	s.
	4·6	Di.	n.
	23·6	Te.	n.
10	13·5	Di.	s.
	22·3	Te.	s.
11	1·3	Rh.	n.
	8·8	En.	n.
	17·4	Mi.	s.
	20·9	Te.	n.
	22·3	Di.	n.
12	16·0	Mi.	s.
	17·7	En.	n.
	19·6	Te.	s.
13	7·1	Di.	s.
	7·5	Rh.	s.
	10·2	En.	s.
	14·6	Mi.	s.
	18·2	Te.	n.
14	13·3	Mi.	s.

1884.	h		
Oct. 14	16·0	Di.	n.
	16·9	Te.	s.
	19·1	En.	s.
15	11·5	En.	n.
	11·9	Mi.	s
	13·7	Rh.	n.
	15·5	Te.	n.
16	0·8	Di.	s.
	10·5	Mi.	s.
	14·2	Te.	s.
	20·4	En.	n.
17	9·7	Di.	n.
	12·8	En.	s.
	12·8	Te.	n.
	19·9	Rh.	s.
	20·4	Mi.	n.
18	11·5	Te.	s.
	18·5	Di.	s.
	19·0	Mi.	n.
	21·7	En.	s.
19	10·1	Te.	n.
	14·1	En.	n.
	17·6	Mi.	n.
20	2·0	Rh.	n.
	3·3	Di.	n.
	6·6	En.	s.
	8·8	Te.	s.
	16·3	Mi.	n.
21	7·4	Te.	n.
	12·2	Di.	s.
	14·9	Mi.	n.
	15·4	En.	s.
22	6·1	Te.	s.
	7·9	En.	n.
	8·2	Rh.	s.
	13·5	Mi.	n.
	21·0	Di.	n.
23	4·7	Te.	n.
	12·1	Mi.	n.
	16·8	En.	n.

1884.	h		
Oct. 24	3·4	Te.	s.
	5.8	Di.	s.
	9·2	En.	s.
	10·7	Mi.	n.
	14·4	Rh.	n.
25	2·0	Te.	n.
	9·3	Mi.	n.
	14·6	Di.	n.
	18·1	En.	s.
26	0·7	Te.	s.
	10·5	En.	n.
	19·3	Mi.	s.
	20·6	Rh.	s.
	23·3	Te.	n.
	23·5	Di.	s.
27	17·9	Mi.	s.
	19·4	En.	n.
	22·0	Te.	s.
28	8·3	Di.	n.
	11·8	En.	s.
	16·5	Mi.	s.
	20·6	Te.	n.
29	2·8	Rh.	n.
	15·1	Mi.	s.
	17·1	Di.	s.
	19·3	Te.	s.
	20·7	En.	s.
30	13·2	En.	n.
	13·7	Mi.	s.
	17·9	Te.	n.
31	2·0	Di.	n.
	5·6	En.	s.
	8·9	Rh.	s.
	12·3	Mi.	s.
	16·6	Te.	s.
Nov. 1	10·8	Di.	s.
	10·9	Mi.	s.
	14·5	En.	s.
	15·2	Te.	n.
2	6·9	En.	n.

1884.	h		
Nov. 2	9·6	Mi.	s.
	13·8	Te.	s.
	15·1	Rh.	n.
	19·6	Di.	n.
3	12·5	Te.	n.
	15·8	En.	n.
4	4·4	Di.	s.
	11·1	Te.	s.
	21·3	Rh.	s.
5	9·8	Te.	n.
	13·3	Di.	n.
6	8·4	Te.	s.
	22·1	Di.	s.
7	3·4	Rh.	n.
	7·1	Te.	n.
	13·9	Mi.	n.
	18·4	En.	n.
8	5·7	Te.	s.
	6·9	Di.	n.
	10·8	En.	s.
	12·5	Mi.	n.
9	4·4	Te.	n.
	9·6	Rh.	s.
	11·2	Mi.	n.
	15·7	Di.	s.
	19·7	En.	s.
10	3·0	Te.	s.
	9·8	Mi.	n.
	12·2	En.	n.
11	0·6	Di.	n.
	1·7	Te.	n.
	4·6	En.	s.
	8·4	Mi.	n.
	15·8	Rh.	n.
	19·7	Mi.	s.
	21·0	En.	n.
12	0·3	Te.	s.
	9·4	Di.	s.
	13·5	En.	s.
	18·3	Mi.	s.

1884.	h		
Nov. 12	22·9	Te.	n.
13	5·9	En.	n.
	16·9	Mi.	s.
	18·2	Di.	n.
	21·6	Te.	s.
	21·9	Rh.	s.
14	14·8	En.	n.
	15·5	Mi.	s.
	20·2	Te.	n.
15	3·0	Di.	s.
	7·2	En.	s.
	14·1	Mi.	s.
	18·9	Te.	s.
16	4·1	Rh.	n.
	11·9	Di.	n.
	12·7	Mi.	s.
	16·1	En.	s.
	17·5	Te.	n.
17	8·5	En.	n.
	11·4	Mi.	s.
	16·2	Te.	s.
	20·7	Di.	s.
18	10·0	Mi.	s.
	10·2	Rh.	s.
	14·8	Te.	n.
	17·4	En.	n.
19	5·5	Di.	n.
	8·6	Mi.	s.
	9·8	En.	s.
	13·4	Te.	s.
	19·9	Mi.	n.
20	12·1	Te.	n.
	14·3	Di.	s.
	16·4	Rh.	n.
	18·5	Mi.	n.
	18·7	En.	s.
21	10·7	Te.	s.
	11·2	En.	n.
	17·1	Mi.	n.
	23·2	Di.	n.

1884.	h		
Nov. 22	9·4	Te.	n.
	15·7	Mi.	n.
	20·0	En.	n.
	22·5	Rh.	s.
23	8·0	Di.	s.
	8·0	Te.	s.
	12·5	En.	s.
	14·3	Mi.	n.
24	4·9	En.	n.
	6·7	Te.	n.
	13·0	Mi.	n.
	16·8	Di.	n.
	21·3	En.	s.
25	4·7	Rh.	n.
	5·3	Te.	s.
	11·6	Mi.	n.
	13·8	En.	n.
26	1·6	Di.	s.
	3·9	Te.	n.
	6·2	En.	s.
	10·2	Mi.	n.
27	2·6	Te.	s.
	8·8	Mi.	n.
	10·4	Di.	n.
	10·8	Rh.	s.
	15·1	En.	s.
28	1·2	Te.	n.
	7·4	Mi.	n.
	7·5	En.	n.
	18·7	Mi.	s.
	19·3	Di.	s.
	23·9	Te.	s.
29	6·0	Mi.	n.
	16·4	En.	n.
	17·0	Rh.	n.
	17·3	Mi.	s.
	22·5	Te.	n.
30	4·1	Di.	n.
	8·8	En.	s.
	15·9	Mi.	s.

1884.	h			1884.	h			1884.	h		
Nov. 30	21·2	Te.	s.	Dec. 10	23·7	Rh.	s.	Dec. 20	0·3	Rh.	s.
Dec. 1	12·9	Di.	s.	11	2·6	Di.	n.		5·5	En.	n.
	19·8	Te.	n.		6·2	Te.	n.		10·8	Mi.	s.
	23·1	Rh.	s.		7·8	En.	s.		16·4	Di.	s.
2	18·4	Te.	s.		12·0	Mi.	n.		16·7	Te.	n.
	21·7	Di.	n.	12	4·9	Te.	s.	21	9·4	Mi.	s.
3	17·1	Te.	n.		10·6	Mi.	n.		14·4	En.	n.
4	5·3	Rh.	n.		11·5	Di.	s.		15·3	Te.	s.
	6·5	Di.	s.		16·7	En.	s.	22	1·2	Di.	n.
	10·4	Mi.	s.	13	3·5	Te.	n.		6·5	Rh.	n.
	11·5	En.	s.		5·9	Rh.	n.		6·8	En.	s.
	15·7	Te.	s.		9·1	En.	n.		8·0	Mi.	s.
5	9·0	Mi.	s.		9·2	Mi.	n.		13·9	Te.	n.
	14·4	Te.	n.		20·3	Di.	n.	23	6·6	Mi.	s.
	15·4	Di.	n.	14	2·2	Te.	s.		10·0	Di.	s.
	20·3	Mi.	n.		7·8	Mi.	n.		12·6	Te.	s.
	20·3	En.	s.		18·0	En.	n.		15·6	En.	s.
6	7·6	Mi.	s.	15	0·8	Te.	n.		18·0	Mi.	n.
	11·4	Rh.	s.		5·1	Di.	s.	24	8·1	En.	n.
	12·8	En.	n.		6·4	Mi.	n.		11·2	Te.	n.
	13·0	Te.	s.		10·4	En.	s.		12·6	Rh.	s.
	18·9	Mi.	n.		12·0	Rh.	s.		16·6	Mi.	n.
7	0·2	Di.	s.		17·7	Mi.	s.		18·8	Di.	n.
	5·2	En.	s.		23·4	Te.	s.	25	9·9	Te.	s.
	6·2	Mi.	s.	16	13·9	Di.	n.		15·2	Mi.	n.
	11·7	Te.	n.		16·4	Mi.	s.		17·0	En.	n.
	17·5	Mi.	n.		19·3	En.	s.	26	3·7	Di.	s.
	21·6	En.	n.		22·1	Te.	n.		8·5	Te.	n.
8	9·0	Di.	n.	17	11·8	En.	n.		9·4	En.	s.
	10·3	Te.	s.		15·0	Mi.	s.		13·8	Mi.	n.
	14·1	En.	s.		18·2	Rh.	n.		18·8	Rh.	n.
	16·1	Mi.	n.		20·7	Te.	s.	27	1·9	En.	n.
	17·6	Rh.	n.		22·7	Di.	s.		7·2	Te.	s.
9	6·5	En.	n.	18	13·6	Mi.	s.		12·4	Mi.	n.
	8·9	Te.	n.		19·4	Te.	n.		12·5	Di.	n.
	14·8	Mi.	n.		20·6	En.	n.		18·3	En.	s.
	17·8	Di.	s.	19	7·6	Di.	n.	28	5·8	Te.	n.
10	7·6	Te.	s.		12·2	Mi.	s.		10·8	En.	n.
	13·4	Mi.	n.		13·1	En.	s.		11·3	Mi.	n.
	15·4	En.	n.		18·0	Te.	s.		21·3	Di.	s.

1884.	h		
Dec. 29	0·9	Rh.	s.
	4·5	Te.	s.
30	3·1	Te.	n.
	6·1	Di.	n.
31	1·7	Te.	s.
	7·1	Rh.	n.
	15·0	Di.	s.
1885.			
Jan. 1	0·4	Te.	n.
	13·4	En.	n.
	16·8	Mi.	s.
	23·0	Te.	s.
	23·8	Di.	n.
2	5·8	En.	s.
	13·2	Rh.	s.
	15·4	Mi.	s.
	21·7	Te.	n.
3	8·6	Di.	s.
	14·0	Mi.	s.
	14·7	En.	s.
	20·3	Te.	s.
4	7·1	En.	n.
	12·6	Mi.	s.
	17·4	Di.	n.
	19·0	Te.	n.
	19·4	Rh.	n.
5	11·2	Mi.	s.
	16·0	En.	n.
	17·6	Te.	s.
6	2·3	Di.	s.
	8·4	En.	s.
	9·9	Mi.	s.
	16·3	Te.	n.
7	0·9	En.	n.
	1·6	Rh.	s.
	8·5	Mi.	s.
	11·1	Di.	n.
	14·9	Te.	s.
	17·3	En.	s.
8	7·1	Mi.	s.

1885.	h		
Jan. 8	9·8	En.	n.
	13·5	Te.	n.
	18·4	Mi.	n.
	19·9	Di.	s.
9	2·2	En.	s.
	5·7	Mi.	s.
	7·7	Rh.	n.
	12·2	Te.	s.
	17·0	Mi.	n.
	18·6	En.	n.
10	4·7	Di.	n.
	10·8	Te.	n.
	11·1	En.	s.
	15·6	Mi.	n.
11	3·5	En.	n.
	9·5	Te.	s.
	13·6	Di.	s.
	13·9	Rh.	s.
	14·2	Mi.	n.
12	8·1	Te.	n.
	12·4	En.	n.
	12·9	Mi.	n.
	22·4	Di.	n.
13	4·8	En.	s.
	6·8	Te.	s.
	11·5	Mi.	n.
	20·1	Rh.	n.
14	5·4	Te.	n.
	7·2	Di.	s.
	10·1	Mi.	n.
	13·7	En.	s.
15	4·1	Te.	s.
	6·2	En.	n.
	8·7	Mi.	n.
	16·1	Di.	n.
16	2·2	Rh.	s.
	2·7	Te.	n.
	7·3	Mi.	n.
	15·0	En.	n.
17	0·9	Di.	s.

1885.	h		
Jan. 17	1·4	Te.	s.
	5·9	Mi.	n.
	7·5	En.	s.
18	0·0	Te.	n.
	4·6	Mi.	n.
	8·4	Rh.	n.
	9·7	Di.	n.
	15·9	Mi.	n.
	16·4	En.	s.
	22·7	Te.	s.
19	8·8	En.	n.
	14·5	Mi.	s.
	18·5	Di.	s.
	21·3	Te.	n.
20	13·1	Mi.	s.
	14·6	Rh.	s.
	17·7	En.	n.
	20·0	Te.	s.
21	3·4	Di.	n.
	10·1	En.	s.
	11·7	Mi.	s.
	18·6	Te.	n.
22	2·6	En.	n.
	10·3	Mi.	s.
	12·2	Di.	s.
	17·3	Te.	s.
	19·0	En.	s.
	20·8	Rh.	n.
23	8·9	Mi.	s.
	11·4	En.	n.
	15·9	Te.	n.
	21.0	Di.	n.
24	3·9	En.	s.
	7·6	Mi.	s.
	14·6	Te.	s.
25	2·9	Rh.	s.
	5·9	Di.	s.
	13·2	Te.	n.
26	11·9	Te.	s.
	14·7	Di.	n.

1885.	h		
Jan. 27	9·1	Rh.	n.
	10·5	Te.	n.
	23·5	Di.	s.
28	6·5	En.	s.
	9·2	Te.	s.
	13 3	Mi.	n.
29	7·8	Te.	n.
	8·4	Di.	n.
	12·0	Mi.	n.
	15·3	Rh.	s.
	15·4	En.	s.
30	6·5	Te.	s.
	7·9	En.	n.
	10·6	Mi.	n.
	17·2	Di.	s.
31	0·3	En.	s.
	5·1	Te.	n.
	9·0	Mi.	n.
	16.7	En.	n.
	21·5	Rh.	s.
Feb. 1	2·1	Di.	n.
	3·8	Te.	s.
	7·8	Mi.	n.
	9·2	En.	s.
2	1·6	En.	n.
	2·4	Te.	n.
	6·4	Mi.	n.
	10·9	Di.	s.
3	1·1	Te.	s.
	3·7	Rh.	s.
	5·1	Mi.	n.
	10·5	En.	n.
	16·4	Mi.	s.
	19·7	Di.	n.
	23·7	Te.	n.
4	2·9	En.	s.
	15·0	Mi.	s.
	19·4	En.	n.
	22·4	Te.	s.
5	4·6	Di.	s.

1885.	h		
Feb. 5	9·9	Rh.	n.
	11·8	En.	s.
	13·6	Mi.	s.
	21·0	Te.	n.
6	4·3	En.	n.
	12·2	Mi.	s.
	13·4	Di.	n.
	19·7	Te.	s.
7	10·8	Mi.	s.
	13·2	En.	n.
	16·1	Rh.	s.
	18·3	Te.	n.
	22·2	Di.	s.
8	5·6	En.	s.
	9·5	Mi.	s.
	17·0	Te.	s.
9	7·1	Di.	n.
	8·1	Mi.	s.
	14·5	En.	s.
	15·6	Te.	n.
	22·3	Rh.	n.
10	6·7	Mi.	s.
	6·9	En.	n.
	14·3	Te.	s.
	15·9	Di.	s.
11	5·3	Mi.	s.
	12·9	Te.	n.
	15·8	En.	n.
12	0·8	Di.	n.
	4·5	Rh.	s.
	8·3	En.	s.
	11·6	Te.	s.
	15·2	Mi.	n.
13	0·7	En.	n.
	9·6	Di.	s.
	10·3	Te.	n.
	13·9	Mi.	n.
14	8·9	Te.	s.
	9·6	En.	n.
	10·7	Rh.	n.

1885.	h		
Feb. 14	12·5	Mi.	n.
	18·5	Di.	n.
15	2·0	En.	s.
	7·6	Te.	n.
	11·1	Mi.	n.
16	3·3	Di.	s.
	6·2	Te.	s.
	9·7	Mi.	n.
	10·9	En.	s.
	16·9	Rh.	s.
17	3·4	En.	n.
	4·9	Te.	n.
	8·3	Mi.	n.
	12·2	Di.	n.
18	3·5	Te.	s.
	7·0	Mi.	n.
	12·3	En.	n.
	21·0	Di.	s.
	23·1	Rh.	n.
19	2·2	Te.	n.
	4·7	En.	s.
	5·6	Mi.	n.
20	0·8	Te.	s.
	4·2	Mi.	n.
	5·8	Di.	n.
	13·6	En.	s.
	23·5	Te.	n.
21	5·4	Rh.	s.
	14·7	Di.	s.
	22·2	Te.	s.
22	20·8	Te.	n.
	23·5	Di.	n.
23	11·6	Rh.	n.
	19·5	Te.	s.
24	8·4	Di.	s.
	10·0	Mi.	s.
	16·3	En.	s.
	18·1	Te.	n.
25	8·6	Mi.	s.
	8·7	En.	n.

1885.	h		
Feb. 25	16·8	Te.	s.
	17·2	Di.	n.
	17·8	Rh.	s.
26	1·2	En.	s.
	7·2	Mi.	s.
	15·5	Te.	n.
	17·6	En.	n.
27	2·1	Di.	s.
	5·9	Mi.	s.
	10·0	En.	s.
	14·1	Te.	s.
28	0·0	Rh.	n.
	2·5	En.	n.
	4·5	Mi.	s.
	10·9	Di.	n.
	12.8	Te.	n.
Mar. 1	11·4	En.	n.
	11·4	Te.	s.
	14·4	Mi.	n.
	19·8	Di.	s.
2	3·8	En.	s.
	6·3	Rh.	s.
	10·1	Te.	n.
	13·0	Mi.	n.
3	4·7	Di.	n.
	8·7	Te.	s.
	11·7	Mi.	n.
	12·7	En.	s.
4	5·2	En.	n.
	7·4	Te.	n.
	10·3	Mi.	n.
	12·5	Rh.	n.
	13·5	Di.	s.
5	6·1	Te.	s.
	8·9	Mi.	n.
	14·1	En.	n.
	22·4	Di.	n.
6	4·7	Te.	n.
	6·5	En.	s.
	7·5	Mi.	n.

1885.	h		
Mar. 6	18·8	Rh.	s.
7	3·4	Te.	s.
	6·2	Mi.	n.
	7·2	Di.	s.
	15·4	En.	s.
8	2·0	Te.	n.
	4·8	Mi.	n.
	7·8	En.	n.
	16·1	Di.	n.
	16·1	Mi.	s.
9	0·3	En.	s.
	0·7	Te.	s.
	1·0	Rh.	n.
	3·4	Mi.	n.
	23·4	Te.	n.
10	0·9	Di.	s.
	9·2	En.	s.
	13·3	Mi.	s.
	22·0	Te.	s.
11	1·6	En.	n.
	7·2	Rh.	s.
	9·8	Di.	n.
	12·0	Mi.	s.
	20·7	Te.	n.
12	10·5	En.	n.
	10·6	Mi.	s.
	18·6	Di.	s.
	19·4	Te.	s.
13	3·0	En.	s.
	9·2	Mi.	s.
	13·5	Rh.	n.
	18·0	Te.	n.
14	3·5	Di.	n.
	7·8	Mi.	s.
	11·9	En.	s.
	16·7	Te.	s.
15	4·3	En.	n.
	6·5	Mi.	s.
	12·4	Di.	s.
	15·3	Te.	n.

1885.	h		
Mar. 15	19·7	Rh.	s.
16	5·1	Mi.	s.
	13·2	En.	n.
	14·0	Te.	s.
	21·2	Di.	n.
17	3.7	Mi.	s.
	5·7	En.	s.
	12·7	Te.	n.
18	2·0	Rh.	n.
	6·1	Di.	s.
	11·3	Te.	s.
	13·6	Mi.	n.
	14·6	En.	s.
19	7·0	En.	n.
	10·0	Te.	n.
	12·3	Mi.	n.
	15·0	Di.	n.
20	8·2	Rh.	s.
	8·7	Te.	s.
	10·9	Mi.	n.
	15·9	En.	n.
	23·8	Di.	s.
21	7·3	Te.	n.
	8·3	En.	s.
	9·5	Mi.	n.
22	6·0	Te.	s.
	8·7	Di.	n.
	14·5	Rh.	n.
23	4·6	Te.	n.
	17·5	Di.	s.
24	3·3	Te.	n.
	20·8	Rh.	s.
25	2·0	Te.	n.
	2·4	Di.	n.
26	0·6	Te.	s.
	11·3	Di.	s.
	23·3	Te.	n.
27	3·6	Rh.	n.
	20·1	Di.	n.
	22·6	Te.	s.

1885.	h			1885.	h			1885.	h		
Mar. 28	20·6	Te.	n.	Mar. 31	22·7	Di.	s.	April 5	1·3	Di.	n.
29	5·0	Di.	s.	April 1	15·3	Te.	n.		4·1	Rh.	n.
	9·3	Rh.	s.	2	7·6	Di.	n.		10·0	Te.	n.
	19·3	Te.	s.		14·0	Te.	s.	6	8·6	Te.	s.
30	13·9	Di.	n.		21·8	Rh.	s.		10·2	Di.	s.
	18·0	Te.	n.	3	12·6	Te.	n.	7	7·3	Te.	n.
31	15·6	Rh.	n.		16·5	Di.	s.		10·4	Rh.	s.
	16·6	Te.	s.	4	11·3	Te.	s.		19·1	Di.	n.

By means of this list of conjunctions, approximate values of the coordinates x and y, expressed in semi-diameters of the planet's equator, may be easily found for any other time t in the following little table, the argument of which is the interval τ between the time t and the time of the nearest preceding or following conjunction.

τ	Mimas.		Enceladus.		Tethys.		Dione.		Rhea.	
h	x_1	y_1	x_2	y_2	x_3	y_3	x_4	y_4	x_5	y_5
0	0·0	1·4	0·0	1·8	0·0	2·3	0·0	2·9	0·0	4·0
1	0·9	1·4	0·8	1·7	0·7	2·2	0·6	2·9	0·5	4·0
2	1·7	1·2	1·5	1·7	1·4	2·2	1·2	2·8	1·0	4·0
3	2·3	0·9	2·2	1·5	2·0	2·1	1·8	2·8	1·5	4·0
4	2·8	0·6	2·8	1·3	2·6	1·9	2·4	2·7	2·0	3·9
5	3·1	0·3	3·3	1·0	3·2	1·7	2·9	2·6	2·5	3·8
6	3·1	0·1	3·7	0·7	3·7	1·5	3·5	2·4	3·0	3·8
7	2·9	0·5	3·9	0·4	4·1	1·3	4·0	2·3	3·5	3·7
8	2·5	0·9	4·0	0·1	4·5	1·0	4·4	2·1	4·0	3·6
9					4·7	0·7	4·8	1·9	4·5	3·5
10					4·9	0·4	5·2	1·7	4·9	3·4
11					5·0	0·1	5·6	1·4	5·3	3·2
12							5·8	1·2	5·7	3·1
13							6·1	0·9	6·1	2·9
14							6·2	0·7	6·5	2·8
15							6·3	0·4	6·8	2·6
16							6·4	0·1	7·1	2·4
20									8·2	1·6
27·1									8·9	0·0

The satellites move in the direction of *de*creasing position-angles. They are at their greatest elongations *Mi.* $5^h{\cdot}7$, *En.* $8^h{\cdot}2$ *Te.* $11^h{\cdot}3$, *Di.* $16^h{\cdot}4$, *Rh.* $27^h{\cdot}1$, before or after the times of conjunction. Observers who are desirous to follow the motions of the satellites will do well to lay down the data of the table graphically on a sufficiently large scale, so that, by marking the

corresponding times for the night of observing, they may get information about the places of the satellites at a glance.

The following ephemerides of the three outer satellites give their positions in reference to the circle of declination. I publish the ephemeris of *Hyperion* not without hesitation, as the Washington observations made during the last three oppositions have not yet become known to me (the last series which I have seen is that of 1880, published in No. 2394 of the *Astron. Nachrichten*), and the available data are not sufficient for such a determination of the orbit as to allow the preparation of accurate predictions four years in advance. Nevertheless I prefer to run the risk of giving a detailed ephemeris, in order, if possible, to induce the possessors of powerful instruments to take part in the observations of *Hyperion* and to contribute accurate data for the proper investigation of the problem of the motion of the satellite.

Differences of Right Ascension and Declination between the Satellites Titan, Hyperion, and Iapetus, and the Centre of Saturn.

	Titan.		*Hyperion.*		*Iapetus.*	
0^h Gr. 1884.	α_6-A s	δ_6-D ″	α_7-A s	δ_7-D ″	α_8-A s	δ_8-D ″
Aug. 31	+11·64	+ 47·3	+10·45	− 72·6	+28·80	+174·0
Sept. 1	+ 8·41	+ 68·6	+13·67	− 52·5	+26·85	+175·7
2	+ 3·82	+ 78·9	+16·03	− 28·9	+24·74	+176·5
3	− 1·42	+ 76·2	+17·41	− 3·5	+22·48	+176·3
4	− 6·43	+ 60·9	+17·71	+ 22·2	+20·07	+175·0
5	−10·40	+ 35·3	+16·90	+ 46·7	+17·54	+172·7
6	−12·70	+ 3·9	+15·00	+ 68·1	+14·90	+169·3
7	−12·98	− 28·2	+12·06	+ 85·0	+12·16	+164·9
8	−11·28	− 56·2	+ 8·19	+ 95·5	+ 9·33	+159·6
9	− 7·89	− 75·9	+ 3·66	+ 98·2	+ 6·43	+153·3
10	− 3·36	− 84·8	− 1·21	+ 92·1	+ 3·49	+146·0
11	+ 1·68	− 81·6	− 5·98	+ 76·5	+ 0·51	+137·8
12	+ 6·50	− 67·0	−10·10	+ 52·4	− 2·49	+128·7
13	+10·42	− 42·8	−13·00	+ 21·9	− 5·48	+118·8
14	+12·85	− 12·4	−14·30	− 11·5	− 8·45	+108·1
15	+13·40	+ 19·9	−13·82	− 43·5	−11·38	+ 96·7
16	+11·91	+ 49·3	−11·68	− 70·5	−14·25	+ 84·7
17	+ 8·58	+ 70·9	− 8·26	− 89·8	−17·04	+ 72·1
18	+ 3·83	+ 81·2	− 3·99	−100·4	−19·73	+ 58·9
19	− 1·57	+ 78·1	+ 0·65	−102·0	−22·30	+ 45·3
20	− 6·75	+ 61·9	+ 5·27	− 95·4	−24·74	+ 31·4
21	−10·80	+ 35·4	+ 9·51	− 81·7	−27·02	+ 17·2
22	−13·11	+ 3·0	+13·13	− 62·4	−29·14	+ 2·9
23	−13·35	− 29·6	+15·91	− 38·9	−31·07	− 11·6
24	−11·54	− 58·6	+17·71	− 13·0	−32·79	− 25·9

	Titan.		*Hyperion.*		*Iapetus.*	
0^h Gr.	α_6-A	δ_6-D	α_7-A	δ_7-D	α_8-A	δ_8-D
1884.	s	″	s	″	s	″
Sept. 25	− 8·01	− 78·6	+18·41	+ 13·9	−34·30	− 40·1
26	− 3·30	− 87·3	+17·97	+ 40·0	−35·59	− 54·2
27	+ 1·90	− 83·6	+16·38	+ 63·6	−36·64	− 67·9
28	+ 6·84	− 68·1	+13·62	+ 82·9	−37·44	− 81·2
29	+10·84	− 42·9	+ 9·88	+ 96·2	−37·99	− 94·0
30	+13·29	− 11·5	+ 5·33	+101·7	−38·28	−106·2
Oct. 1	+13·78	+ 21·8	+ 0·29	+ 98·2	−38·31	−117·7
2	+12·17	+ 51·8	− 4·79	+ 84·9	−38·08	−128·4
3	+ 8·63	+ 73·7	− 9·35	+ 62·2	−37·59	−138·3
4	+ 3·69	+ 83·7	−12·83	+ 31·8	−36·84	−147·3
5	− 1·88	+ 79·9	−14·68	− 2·2	−35·83	−155·4
6	− 7·17	+ 62·6	−14·71	− 36·3	−34·57	−162·4
7	−11·28	+ 34·9	−12·96	− 66·2	−33·07	−168·3
8	−13·56	+ 1·3	− 9·75	− 88·8	−31·35	−173·1
9	−13·70	− 32·6	− 5·51	−102·4	−29·40	−176·8
10	−11·73	− 61·6	− 0·75	−106·7	−27·25	−179·3
11	− 8·01	− 81·6	+ 4·09	−102·2	−24·91	−180·6
12	− 3·11	− 89·9	+ 8·65	− 89·9	−22·39	−180·7
13	+ 2·25	− 85·4	+12·62	− 71·3	−19·72	−179·6
14	+ 7·31	− 68·8	+15·79	− 48·0	−16·90	−177·4
15	+11·35	− 42·4	+17·96	− 21·6	−13·96	−174·0
16	+13·76	− 9·7	+19·02	+ 6·3	−10·92	−169·4
17	+14·13	+ 24·4	+18·90	+ 33·9	− 7·80	−163·7
18	+12·35	+ 54·9	+17·55	+ 59·3	− 4·61	−156·9
19	+ 8·59	+ 76·8	+15·01	+ 80·9	− 1·38	−149·0
20	+ 3·43	+ 86·2	+11·38	+ 96·6	+ 1·87	−140·2
21	− 2·31	+ 81·3	+ 6·83	+104·6	+ 5·12	−130·5
22	− 7·69	+ 62·7	+ 1·67	+103·5	+ 8·35	−119·9
23	−11·81	+ 33·6	− 3·66	+ 92·3	+11·54	−108·5
24	−14·01	− 1·1	− 8·60	+ 71·1	+14·67	− 96·4
25	−13·99	− 35·8	−12·54	+ 39·9	+17·71	− 83·7
26	−11·82	− 65·1	−14·92	+ 6·5	+20·66	− 70·4
27	− 7·88	− 84·8	−15·42	− 29·3	+23·49	− 56·6
28	− 2·78	− 92·3	−14·02	− 61·7	+26·19	− 42·4
29	+ 2·73	− 86·7	−11·01	− 87·1	+28·74	− 27·8
30	+ 7·87	− 68·8	− 6·83	−103·6	+31·12	− 13·1
31	+11·90	− 41·0	− 1·99	−107·8	+33·32	+ 1·7
Nov. 1	+14·22	− 7·3	+ 3·05	−107·7	+35·33	+ 16·7

The rest will be published in the Supplementary Number.

Ephemeris of the Satellite of Neptune, 1884–5. By A. Marth.

P, angle of position of the minor axis of the satellite's apparent orbit, in the direction of superior conjunction.

a, b, major and minor semi-axes of the apparent orbit.

$u-U$, longitude of the satellite in its orbit, reckoned from the point which is in superior conjunction with the planet or in opposition to the Earth.

$U+180°$, planetocentric longitude of the Earth, reckoned in the satellite's orbit from the ascending node on the celestial equator.

B, planetocentric latitude of the Earth above the plane of the satellite's orbit.

0^h Gr.	P	a	b	$u-U$	Diff.	U	B
1884.	°	″	″	°	°	°	°
Sept. 5	320·51	16·61	7·55	222·56	612·53	135·66	27·04
15	320·45	16·70	7·59	115·09	·48	135·75	27·02
25	320·34	16·78	7·61	7·57	·43	135·89	26·97
Oct. 5	320·21	16·85	7·62	260·00	·38	136·07	26·90
15	320·04	16·90	7·62	152·38	·34	136·30	26·81
25	319·86	16·94	7·61	44·72	·30	136·55	26·70
Nov. 4	319·66	16·97	7·59	297·02	·29	136·82	26·59
14	319·45	16·97	7·56	189·31	·28	137·10	26·47
24	319·24	16·96	7·53	81·59	·29	137·39	26·34
Dec. 4	319·05	16·93	7·48	333·88	·29	137·66	26·22
14	318·87	16·89	7·43	226·17	·32	137·90	26·11
24	318·72	16·83	7·38	118·49	·36	138·11	26·01
1885.							
Jan. 3	318·60	16·75	7·33	10·85	·40	138·27	25·93
13	318·51	16·67	7·27	263·25	·45	138·39	25·87
23	318·47	16·58	7·22	155·70	·50	138·45	25·83
Feb. 2	318·46	16·48	7·18	48·20	·56	138·46	25·82
12	318·49	16·39	7·14	300·76	·62	138·41	25·83
22	318·57	16·30	7·11	193·38	·67	138·30	25·86
Mar. 4	318·69	16·21	7·09	86·05	·73	138·13	25·92
14	318·84	16·13	7·07	338·78	612·78	137·92	26·01
24	319·02	16·05	7·07	231·56		137·66	26·11

The values of P, a, b, and $u-U$ are to be interpolated for the times for which the apparent positions of the satellite are required, and the position-angles p and distances s are then found by means of the formulæ

$$s \sin (P-p) = a \sin (u-U)$$
$$s \cos (P-p) = b \cos (u-U).$$

The satellite moves in the direction of decreasing position-angles, and will be at its greatest elongations and at its *superior* and *inferior* conjunctions with the planet at the following hours, G.M.T. :—

	nf. Elong. $p=$ P+90° $s=$ a		*Sup.* Conj. P b		*sp.* Elong. P−90° a		*Inf.* Conj. P+180° b	
1884.		h		h		h		h
Sept.	5	18·6	Sept. 7	5·8	Sept. 8	17·1	Sept. 10	4·4
	11	15·6	13	2·9	14	14·2	16	1·4
	17	12·7	19	0·0	20	11·2	21	22·5
	23	9·8	24	21·0	26	8·3	27	19·6
	29	6·8	30	18·1	Oct. 2	5·4	Oct. 3	16·6
Oct.	5	3·9	Oct. 6	15·2	8	2·5	9	13·7
	11	1·0	12	12·3	13	23·6	15	10·8
	16	22·1	18	9·4	19	20·7	21	7·9
	22	19·2	24	6·5	25	17·8	27	5·0
	28	16·3	30	3·6	31	14·9	Nov. 2	2·1
Nov.	3	13·4	Nov.	0·7	Nov 6	12·0	7	23·2
	9	10·5	10	21·8	12	9·1	13	20·4
	15	7·6	16	18·9	18	6·2	19	17·5
	21	4·7	22	16·0	24	3·3	25	14·6
	27	1·8	28	13·1	30	0·4	Dec. 1	11·7
Dec.	2	23·0	Dec. 4	10·2	Dec. 5	21·5	7	8·8
	8	20·1	10	7·3	11	18·6	13	5·9
	14	17·2	16	4·5	17	15·7	19	3·0
	20	14·3	22	1·6	23	12·8	25	0·1
	26	11·4	27	22·7	29	9·9	30	21·2
1885.								
Jan.	1	8·5	Jan. 2	19·7	Jan. 4	7·0	Jan. 5	18·3
	7	5·6	8	16·8	10	4·1	11	15·4
	13	2·6	14	13·9	16	1·2	17	12·5
	18	23·7	20	11·0	21	22·3	23	9·5
	24	20·8	26	8·1	27	19·3	29	6·6
	30	17·8	Feb. 1	5·1	Feb. 2	16·4	Feb. 4	3·6
	5	14·9	7	2·2	8	13·4	10	0·7
	11	11·9	12	23·2	14	10·5	15	21·7
	17	9·0	18	20·2	20	7·5	21	18·8
	23	6·0	24	17·3	26	4·5	27	15·8
	1	3·0	Mar. 2	14·3	Mar. 4	1·5	Mar. 5	12·8
	7	0·1	8	11·3	9	22·6	11	9·8
	12	21·1	14	8·3	15	19·6	17	6·8
	18	18·1	20	5·3	21	16·6	23	3·8